Institute of Actuaries of India

ACET June 2018 Solutions

Mathematics

1. B. Using simplifications $\left|\vec{a}+\vec{b}\right|^2 = \left|\vec{a}+\vec{b}\right| \cdot \left|\vec{a}+\vec{b}\right| = |\vec{a}|^2 + \left|\vec{b}\right|^2 + 2\vec{a}\cdot\vec{b}$, and (likewise) $\left|\vec{a}-\vec{b}\right|^2 = |\vec{a}|^2 + \left|\vec{b}\right|^2 - 2\vec{a}\cdot\vec{b}$ we get $\vec{a}\cdot\vec{b} = 0$. Hence the conclusion.

2. D. The reason is that $\lim_{x\to 0} \sin\frac{1}{x}$ does not exist.

3. D. Here the integrand is an odd function of x.

4. A. Over $(-2,2)$, $|x-1|$ is not differentiable at $x=1$, $|x+1|$ is not differentiable at $x=-1$ and $\tan x$ is not differentiable at $x=\pm\pi/2$.

5. B. $YX = Y \Rightarrow (YX)Y = Y^2 \Rightarrow Y(XY) = Y^2 \Rightarrow YX = Y^2 \Rightarrow Y = Y^2$. As $XY = X$ and $YX = Y$.

6. D. The value of the coefficient determinant is $a(a-1)(a+2)$. So $a \neq 0,1,-2$ for a unique solution.

7. C. Let b be the base of the logarithm. Then both $\log_e x$ and $\log_{10} x$ are strictly increasing functions of x. However, $\log_{1/3} x = \frac{\log_e x}{\log_e(1/3)} = -\frac{\log_e x}{\log_e 3}$ is a strictly decreasing function. (In fact, for any $0 < b < 1$, $y = \log_b x$ is a strictly decreasing function). So 1/3 is the only appropriate base (among the available choices) for the given implication to hold.

8. D. $x^2 + y^2 \leq 4$ implies integral values of x are -2, -1, 0, 1, 2.

9. B.

10. A. Here $\alpha^3 = 1, \beta^3 = 1, 1 + \alpha + \beta = 0$. Hence value of the given expression is -7.

11. A. From Venn diagram, it is clear that $n(X-Y) = 45, n(X\cap Y) = 15$, $n(Y-X) = 65$. Hence $n(X\cup Y) = 45 + 15 + 65 = 125$.

12. B. $f(2x) = 2f(x)$ implies that $x = 0$. So, X is a singleton set.

13. C. Since $\frac{\sqrt{3}+i}{2} = e^{i\pi/6}$ and $\frac{\sqrt{3}-i}{2} = e^{-i\pi/6}$, the given equation implies $e^{in\pi/6} = e^{-in\pi/6}$, i.e., $e^{in\pi/3} = 1$. This holds when n is a multiple of 6. So the least value of the positive integer n is 6.

14. B. The given equation can be factored as $(z-1)(z^3-1) = 0$, which has root $z = 1, 1, \omega, \omega^2$, where ω is one of the imaginary cube roots of unity. The polar representation of the distinct roots are e^{i0}, $e^{i2\pi/3}$ and $e^{i4\pi/3}$ which are the vertices of an equilateral triangle.

15. B. Clearly $\alpha + \beta = 1$, $2\alpha + \beta = 3$. Thus $= 2$, $\beta = -1$. Other ordered pairs also satisfy $g(x) = 2x - 1$.

16. A. $x - [x] \geq 0, \forall x$, which implies that $g(x) \geq 1 > 0$. Thus $f(g(x)) = 1\ \forall x$.

17. D. The given determinant is equal to $(x-1)^3$.

18. A. $n(F) = 38$, $n(B) = 15$, $n(C) = 20$, $n(F \cup B \cup C) = 58$, $n(F \cap B \cap C) = 3$, where F, B and C are sets of men who received medals in Football, Basketball and Cricket respectively. $n(F \cap B) + n(F \cap C) + n(B \cap C) = 3 + 38 + 15 + 20 - 58 = 18$. Required number $=$ $n(F \cap B) + n(F \cap C) + n(B \cap C) - 3n(F \cap B \cap C) = 18 - 3 \times 3 = 9$.

19. D. $f'(x) = 2kx - k^2$. This is equal to zero at $x = k/2$. For this to be equal to -2, k should be -4. Also, $f''(x) = 2k$, which is negative for $k = -4$. Therefore, $x = -2$ is only a maxima (not a minima).

Statistics

20. C. By using the formulae $(n_1 + n_2)\mu = n_1\mu_1 + n_2\mu_2$ and $(n_1 + n_2)(\sigma^2 + \mu^2) = n_1(\sigma_1^2 + \mu_1^2) + n_2(\sigma_2^2 + \mu_2^2)$, we have $\sigma_2 = 4$.

21. B. $(50 \times 300 - 48 - 35 + 84 + 53)/300 = 50.18$

22. B. $P(A \cap B) = 2/5$, $P(A|B) = 2/3$. $P(B') = 1 - P(B) = 1 - \frac{P(A \cap B)}{P(A|B)} = 1 - \frac{\frac{2}{5}}{\frac{2}{3}} = \frac{2}{5}$.

23. A. The total number of pairs of different outcomes (while distinguishing between the two die) is 30. The favourable outcomes are (1,2), (2,1), (1,3), (3,1). Therefore, the required probability is 2/15.

24. A. $Corr(X, Y) = \frac{Cov(X,Y)}{\sigma_X \sigma_Y}$. Therefore, $\sigma_Y = \frac{Cov(X,Y)}{\sigma_X Corr(X,Y)} = \frac{4.8}{3 \times 0.6}$.

25. D. $Cov\left(\frac{X}{\sigma_1}+\frac{Y}{\sigma_2},\frac{X}{\sigma_1}-\frac{Y}{\sigma_2}\right)=Var\left(\frac{X}{\sigma_1}\right)-Var\left(\frac{Y}{\sigma_2}\right)+Cov\left(\frac{Y}{\sigma_2},\frac{X}{\sigma_1}\right)-Cov\left(\frac{X}{\sigma_1},\frac{Y}{\sigma_2}\right)=1-1+\rho-\rho=0$. Since the covariance is zero, the correlation is also zero.

26. C. $\log(XY)=\log X+\log Y$ ~normal. Likewise, $\log(X/Y)=\log X-\log Y$ ~normal. Thus, both XY and X/Y have log-normal distribution.

27. B. $P(X<1|X<2)=\frac{P(X<1)}{P(X<2)}=\frac{1-e^{-0.5}}{1-e^{-1}}$.

28. C. Regression coefficient of Y on X, $b_{YX}=8/10=4/5$. Regression coefficient of Y on X, $b_{XY}=18/40=9/20$. Therefore, $r^2=b_{YX}\times b_{XY}=\frac{4}{5}\times\frac{9}{20}=\frac{9}{25}$. Thus, $r=\pm\frac{3}{5}$. Since both the regression coefficients are positive, we take $r=0.6$.

29. A. $r^2=b_{YX}\times b_{XY}=k\times 4=4k$. Therefore, $0\le 4k\le 1$.

30. A. Suppose N_i is a binary variable indicating whether the ball drawn in the i^{th} draw is white. $E(N_i)=P(N_i=1)=\frac{k}{k+m}$. Therefore, the required expected value is $E(N_1+\cdots N_x)=xE(N_1)=\frac{xk}{k+m}$.

31. B. Let the mean of the distribution be λ. Then $P(X=3)=\frac{e^{-\lambda}\lambda^3}{3!}$ and $P(X=4)=\frac{e^{-\lambda}\lambda^4}{4!}$, so that $\frac{P(X=3)}{P(X=4)}=\frac{2}{3}$ means $\frac{e^{-\lambda}\lambda^3}{3!}/\frac{e^{-\lambda}\lambda^4}{4!}=\frac{2}{3}$, i.e., $4/\lambda=2/3$, i.e., $\lambda=6$.

32. C. $\frac{P(X=2)}{P(X\text{ is even})}=\frac{\log 2}{\log 2+\log 4+\cdots+\log(2n)}=\frac{\log 2}{\log(2\times 4\times\cdots\times 2n)}=\frac{\log 2}{\log(2^n\times n!)}=\frac{\log 2}{n\log 2+\log(n!)}$.

Data Interpretation

33. B. Total no. of students 1512. $60\%\times(902+320)+70\%\times(170+120)=936$. Combined pass % is $\frac{936}{1512}=62\%$ (approx.).

34. D. % of girl engineers doing business management $=\frac{50}{170}=29.4\%$.

35. B. The extremes occur in 2010 and 2008; ratio $=\frac{60+50}{40+20}=\frac{11}{6}$.

36. C. The steepest percentage difference occurred in 2009, when it was $\frac{50-40}{40}=25\%$.

37. A. Total Indian tourists= $30/(15\%\times 20\%)=1000$ lakh. Therefore, the number of Indian tourists in the age bracket 40-49 = $25\%\times 1000=250$ lakh.

38. B. $1000\times 20\%\times 55\%=110$ lakh.

English

39.	D
40.	C
41.	A
42.	C
43.	B
44.	D
45.	B
46.	D
47.	A
48.	D
49.	B
50.	A
51.	D
52.	D
53.	B
54.	A
55.	B
56.	C
57.	D
58.	C
59.	B
60.	D
61.	C
62.	B

Logical reasoning

63. C

64. A

65. C .

Number of odd days from the year 2006 to the year 2009 = $(1 + 1 + 2 + 1) = 5$ days.
On 1st January 2006 it was Sunday.
Thus, on 1st Jan 2010, it is Friday.

66. B

As DAUGHTER	TERDAUGH
1 2 3 4 5 6 7 8	6 7 8 1 2 3 4 5

Similarly	APTITUDE	UDEAPTIT
	1 2 3 4 5 6 7 8 6	7 8 1 2 3 4 5

67. C. The correct order is

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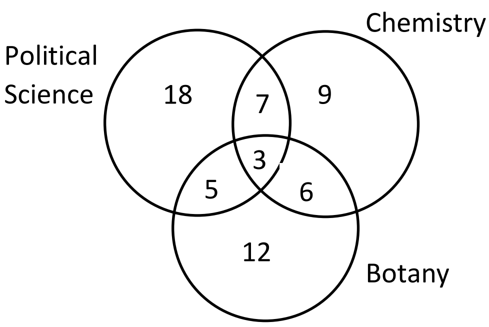
5 6 2 3 4 1

68. B. Dodge, duck and avoid are all synonyms meaning *evade*. Flee means to run away from.

69. B



70. D. The set of fair people contains the set of honest people, which contains the set of politicians. Therefore, some fair people are politicians. By the same logic, some honest persons are politicians too. The other statements do not follow.