# Institute of Actuaries of India

## Subject SP6 – Financial Derivatives Principles

# March 2021 Examination

# INDICATIVE SOLUTION

**Introduction**

The indicative solution has been written by the Examiners with the aim of helping candidates. The solutions given are only indicative. It is realized that there could be other points as valid answers and examiner have given credit for any alternative approach or interpretation which they consider to be reasonable

**Solution 1:**

**i)** Let's evaluate both the transactions.

**Selling the stock**

- Receives Rs. (650 * (1-1%)) = Rs. 643.50

- Loss of dividend of Rs. 45 as on 15th October 2015 whose value as on 1st October 2015 is
  - Rs. 44.90 (if computed @ 5% for 15 days)
  - Rs. 44.89 (if computed @ 6% for 15 days)

  Since the client will be net lender rather than borrower the applicable rate of interest is 5%. And hence the loss of dividend in present value terms should be Rs. 44.90

**Buying the futures**

- Pay the brokerage: Rs. (594 * 0.005) = Rs. 2.97

- Pay the margin: Rs. (651 * 0.05) = Rs. 32.55

- Pay Rs. (594 – 32.55) = Rs. 561.45 on maturity (31st October 2015) and get the shares.

- The value of this amount as on 1st October 2015 is the present value of Rs. 561.45 @ 5% for 31days = Rs. 559.08

In the nutshell, the expected gain from the strategy today is:

Rs. (643.50 – Rs. 44.90 – Rs. 2.97 – Rs. 32.55 – Rs. 559.08) = Rs. 4.00

Hence, there is a minor gain associated with effecting the transactions and can be considered subject to risks in part (ii)

**[9]**

**ii)** The risks in implementing this strategy are:

- The discount is not as substantial as you may infer by looking at the prices and the potential gain is too low considering the many uncertainties associated with the transactions.
- Even if the prices are known for calculating the gain/loss before putting the order on the exchange, the actual transacted price may be different compared to the price used in calculating the gains.
- The company may either reduce or increase the amount of dividend after the transactions. The decrease is beneficial under the strategy but not the increase. Even if the dividend is slightly more than the expected dividend it can eat away a large part of the transaction gains.
- Taxes have been ignored in the above calculations and they may decrease/increase the benefits.
- The daily settlements have been ignored in the above calculations. This may give rise to further cash-flow management problems and interest gain/losses on the same. Exchanges may also change the margin requirement any time potentially impacting the results especially when the expected gain is so low.

**[5]**

**[14 Marks]**

**Solution 2:**

**i)**

Value of the put using Black-Scholes is

$$k_p\, e^{-rT} N(-d_2) - SN(-d_1)$$

where $S$ = Rs. 50m (50% of Rs. 100m), $k_p$ = Rs. 40m

$d_1 = (1/\sigma\sqrt{T}) * ((\ln(S/k_p) + ((r + (1/2)\sigma^2)T) = (1/(0.25 * \sqrt{1}))*((\ln(50/40))+(0.02 +(0.25^2/2))) = 1.09757$
$d_2 = d_1 - \sigma\sqrt{T} = 1.09757 - (0.25 * \sqrt{1}) = 0.84757$

Thus, $N(-d_1) = 0.13620$ and $N(-d_2) = 0.19834$

So, the value of the put = Rs. ((40 * e $^{-0.02}$ * 0.1983) – (50 * 0.1362) ≈ Rs. 966,500 [3]

**ii)** Delta of the put option is $-S\,N(-d_1) = -50 * 0.13620 =$ – Rs. 6,810,000
Delta of the (shorted) call is $-S\,N(d_1)$
Where,
$d_1 = (1/\sigma\sqrt{T}) * ((\ln(S/k_C) + ((r + (1/2)\sigma^2)T)$, with $k_C$ = Rs. 62 m and $\sigma = 0.2$

Thus, in this case, $d_1 = -0.87556$ and $N(d_1) = 0.19063$.

So, delta of (shorted) call is – Rs. 50m * 0.1906 = –Rs. 9,532,000

So, effective equity investment in fund is Rs. (50m – 6.810m – 9.532m) = Rs. 33.658m and the effective equity exposure = 33.658% [5]

**iii)**

(a) If equity prices increase, effective equity exposure (EEE) reduces further and would tend to 0% as the index increases. This is due to the call becoming increasingly in the money, thus making the combined equity / derivative portfolio look more like a bond paying Rs. 64m at time $t = 1$.

(b) If implied volatilities increase, EEE reduces due to the increase in time value of the options.

(c) If a shorter term is used, EEE increases due to the reduced time value of the options.

(d) If the collar is widened, EEE increases because the option values decrease (further out of the money) and hence their delta offset reduces. [4]

**iv)** Rho of the (shorted) call is: - Rs. 62m * 1 * $e^{-0.02}$ * $N(d_2)$; where $d_2 = d_1 - \sigma\sqrt{T} = -0.87556 - 0.2 = -1.0756$;
Hence, $N(d_2)$ is 0.14106.

Thus, rho is –Rs. 62m * 1 * $e^{-0.02}$ * 0.14106 = – Rs. 8,572,700.

Rho of the put is:
– Rs. 40m * 1 * $e^{-0.02}$ * $N(-d_2)$ = –Rs. 40m * 1 * $e^{-0.02}$ * 0.19834 = –Rs. 7,776,500.

Total rho = –Rs. 8,572,500 – Rs. 7,776,500 = – Rs. 16,349,000.
The cash part of the fund is Rs. 50m, and the duration of a zero-coupon bond of maturity $t$ is $-t$; so effective bond duration = (16,349,000 / 50m) = 0.327 years ≈ 4 months

[5]

**v)**

- **Volatility risk – vega:** Changes in the value of the fund's assets as a result of changes in market implied volatilities. An increase in volatilities will increase the values of the put and the shorted call. While these are offsetting, this is still a risk that needs to be understood, especially if there are skew effects between strikes.

- **Volatility risk – gamma:** Non-linear changes in the value of the fund's assets as a result of market movements. A change in equity prices will alter the effective equity exposure.
- **Counterparty risk:** the risk of the derivative counterparty defaulting on its obligations at a point in time when the fund has positive credit exposure to them.

[3]

**[20 Marks]**

**Solution 3:**

**i)**

a) **Desirable features** are:

- Reasonable dispersion of rates over time (due to the Brownian motion), otherwise there will be too large a probability of getting an absurdly high or low value. Market prices do not allow for these extremes [*another way of expressing this is to say that, when rates go too high or low, they tend to revert back to some middle level ( mean-reverting )*]
- No negative interest rates
- Easy to use and calculate, especially when calibrating to market prices bond (and/or swap) prices should be reproduced by the model
- Forward rates should be imperfectly correlated although, this is only important when pricing certain types of option (e.g. yield spread options)
- Volatility of rates of different maturity should be different, with generally shorter rates being the more volatile

b) **Good points** for CIR are that it satisfies most of the desirable features above:

- it is easy to use (algebraic)
- it has time-dependent volatility

But:

- it has no de-correlation of forward rates
- it scores less well on the possible shapes of yield curve it can fit precisely (in fact, only simple upward sloping, simple downward sloping and single-humped are possible)

c) **Over-wide dispersion** is contained by using the explicit mean reversion parameter in the drift term. $\mu$ is the long-term target for $r$ and $\alpha$is the speed (or force) of correction.

**Negative rates** are avoided by making the volatility term diminish as $r$ approaches zero, so provided $\sigma$ is not too large in relation to $\alpha$ and $\mu$.

[8]

**ii)** Let $R(\tau)$ be the $\tau$ period maturity spot rate for any given time $t$.
Then,

$$B(\tau) = e^{-R(\tau).\tau}$$

or, equivalently,

$$R(\tau) = \frac{1}{\tau}\ln B = -\frac{a - br}{\tau}$$

Now,

$$a = c_3\left(ln(c_1) + c_2\tau - ln\left(c_2(\exp(c_1\tau) - 1)\right)\right) \rightarrow c_3\left(c_2\tau - ln\left(c_2\tau - ln\left(c_2\left(exp(c_1\tau)\right)\right)\right)\right).$$

As $\tau \rightarrow \propto$, terms in $\tau$ dominate the constant terms. Thus,

$$a \rightarrow c_3(c_2\tau - \ln c_2 - c_1\tau) \rightarrow c_3(c_2 - c_1)\tau.$$

Also, $b \rightarrow \frac{1}{c_2}\ as\ \tau \rightarrow \infty$. Further, for a given t, r(t) is constant and thus

$$-\frac{br}{\tau} \rightarrow 0\ as\ \tau \rightarrow \infty.$$

Hence,

$$R(\tau) = -\frac{a - br}{\tau} \rightarrow c_3(c_1 - c_2)\ as\ \tau \rightarrow \propto.$$

This is a positive constant, say *R*. Substituting this in the equation,

$$B(\tau) = e^{-R(\tau).\tau}$$

gives us the result.

[5]

**iii)**

**No-arbitrage** models are a class of models which allow recovery of market prices of one set of securities given prices of another set. This gives a world of relative pricing. In a non-arbitrage-free model, securities could be priced using the model and then traded at a different price in the real world, leading to persistent profit. In simplest terms, no-arbitrage is the absence of a free lunch.

No-arbitrage is very important in yield curve models, since most complex structures are limiting cases of simpler structures (such as swaps, caps, floors) and hence ideally the model should recover the prices of the latter exactly.

Also, hedging is done using the simpler structures, so the absence of no – arbitrage would mean the accounting process would be distorted by imaginary gains and losses.

[3]

**[16 Marks]**

**Solution 4:**

**i)** **Structured products** are pre-packaged investments that normally include assets linked to interest plus one or more derivatives. They are generally tied to an index or basket of securities, and are designed to facilitate highly customized risk-return objectives.

Requirements

- Counterparty to structure the product with different characteristics.
- Modelling approach to price the product as market price may not exist.

Risk

- Products are complex in nature not well understood.
- Would be illiquid and have to be tailored for individual investor requirements

[6]

**ii)** The swap can be composed of ordinary swap and daily binary options (Cash or nothing) in order to cancel the fixed payment for that day.

The valuation would be of the swap – value of binary option based on the market level, which need to sold by the holder of the swap.

There would be 2 options one to cancel the fixed payment if the rate goes below 6% and second if the payment goes above 7%

The binary options would be valued using Black formula using the term $N(d_2)$

Simplification: Doing daily calculation would be very complex hence binary options can be simplified by combining it to be weekly or fortnightly.

The value of the binary options are:

$D(S0,T,K,\sigma)=-dC(S0,T,K,\sigma)dK$ [3]

**iii)** A digital option price has the **same shape** (though not the same value) as the corresponding call option delta $(d1)$N(d1).

This is delta for the normal call options hence delta for the binary option would have shape like the gamma for the normal options.

The gamma of binary option is rate of change of delta. The gamma increases dramatically near the boundaries as the changes in delta become high given the option would either be paying or non paying at the boundary. The theoretical gamma at the boundary is infinity. [4]

**iv)** The additional option would make the market level of interest rate irrelevant given the strike price could be moved as per the owners chose.

This would result in randomness in the pricing which cannot be captured using traditional pricing techniques hence a more complex model such as two factor Hull – white model would be required with monte carlo approach over the period of the options.

This would also increase the pricing error hence would require higher compensation to the writer of the option. [3]

**[16 Marks]**

**Solution 5:**

**i)** Margin rate is equity divided by the stock value

| | |
|---|---|
| Money invested | 1,00,00,000.00 |
| Margin rate | 80% |
| Stock price | 10000 |
| total Amount available | 1,25,00,000 |
| Share purchased | 1,250.00 |
| loan amount | 25,00,000.00 |

[2]

**ii)** For margin call below 50% = equity / stock value

Margin rate = (S* number of shares- loan)/S* number of shares

S* No of share* Margin rate = S* number of share – loan

Loan = S ( No of share- No of share * Margin rate)

S= Loan /( No of Share – No of Share * Margin rate)

Substituting in above we get 4000

Investor equity = Shareprice * number of Shares -loan

Equity at the margin call 25,00,000

[4]

**iii)** return on stock = (end price – start price )/ Start price

return on stock -60.0%

return of equity = end equity/start equity – 1

=-75%

The ratio of the return is 80% which is equivalent to the initial margin [1]

[3]

**iv)** Total assets = Share price * number of share + dividend * number of share

Total liability = loan * ( 1+ interest rate)

Equity = total asset – liability

Return = end equity/start equity - 1

| | |
|---|---|
| Stock price | 7000 |
| Dividend | 350 |
| dividend amount | 4,37,500 |
| Stock value + Dividend | 91,87,500 |
| interest | 10% |
| Liability | |
| loan | 27,50,000.00 |
| Equity | 64,37,500.00 |
| return on equity | -0.35625 |

[4]

**v)** Share = (Invested / Margin )* price

| **Short position** | |
|---|---|
| equity | 10000000 |

| | |
|---|---|
| price | 10000 |
| margin | 50% |
| shares | 2000 |

Margin rate = equity / value of stock

(Total asset - Share X price)/( share x P )

Total asset = Share + margin = 2000 x 10000 + 1 cr = 3 cr. [ given shorted share hence received the cash]

Rate when margin is 30%

Margin = (Total Asset – Share x Price )/Share x Price

Rearranging for the price

P = total asset /( Share( 1+ MR) ]. = 3cr / ( 1+ .3) * 2000 = 11538.5

Stock return = 11538.5/10000 c. 15%

Equity return = current equity/initial investment – 1

Current equity = Total Asset – share price x number of shares = 3cr – 11538.5x2000 = 69,23,076.92

Expected loss = 1cr - 69,23,077 = 30,76,923

Return on equity = - 31%

[5]

**[18 Marks]**

**Solution 6:**

**i)** Advantages

IBOR could be calculated in many different timeframes ranging from overnight to as long as 12 months. The rates are widely available hence easier to use. There is low impact of liquidity on the rates given they are not based on actual transaction.

Disadvantages

bankers at several major financial institutions can collude with each other to manipulate the rates which can result in low confidence in the financial systems. The key driver for a healthy market is the confidence on the reliability of the base rate on which the spread for credit risk is calculated. If the base rate can be manipulated by bigger players for their profit then it can be catastrophic for the financial system.

[3]

**ii)** These based on transactions and is seen as preferable to IBOR since it is based on data from observable transactions rather than on estimated borrowing rates hence it is less prone to manipulation.

Better alignment of the rate to market instruments as the SOFR is used by the market participants in financial instruments.

Disadvantages

Liquidity: There may be challenges in the markets where market transactions are low difficult to determine stable SOFR.

SOFR is only an overnight rate which is a serious drawback when compared to IBOR. This is because IBOR could be calculated in many different timeframes ranging from overnight to as long as 12 months.

[3]

**[6 Marks]**

**Solution 7:**

**i)** Callable bonds are bonds which has embedded options by which borrower has option to pay the full redemption amount before the maturity date of the bond. The forward rate and price volatility at call date is volatility of price and rate at future period. The volatility of rate and price are approximately related by the following equation.

Volatility price = duration x forward yield x forward yield volatility [3]

**ii)** Let us first estimate the value of the bond without the option.

| | | | |
|---|---|---|---|
| Non callable bond price | | | |
| Nominal | 100 | | |
| Duration | 5 | | |
| Continous compounded yield | 7% | | |
| coupon | 5 | | |
| Risk free rate | 5% | | |
| | coupon | discount | discounted value |
| 1 | 5 | 0.93 | 4.66 |
| 2 | 5 | 0.87 | 4.35 |
| 3 | 5 | 0.81 | 4.05 |
| 4 | 5 | 0.76 | 3.78 |
| 5 | 5 | 0.70 | 3.52 |
| 5 | 100 | 0.70 | 70.47 |
| | | | |
| Price of bond ( sum of the discounted value | 90.83 | | |

We have to use blacks model to estimate the price of the option. Let us estimate each parameter to be used in the blacks model

1) Forward price of the bond
   This would be : Current price X ( 1+ risk free rate ). + coupon
   Assumption coupon is paid at the end of year 1

   Price at Year. 1 90.5

2) Other variables
   a. Duration. = 80% x 4.5 (as duration after 1 year would be approx. 80% of the current duration
   b. Forward price volatility = duration x yield x yield volatility
   c. Strike price = 100 – 5 = 95

d. Risk free rate = 5%

e. Value of option is

$$C = \exp(-rt)[F\,\mathbf{N}(d_1) - X\,\mathbf{N}(d_2)]:$$

Where :

$$d_1 = \frac{\ln\left(F/X\right) + \frac{1}{2}\sigma^2 t}{\sigma\sqrt{t}}$$

$$d_2 = \frac{\ln\left(F/X\right) - \frac{1}{2}\sigma^2 t}{\sigma\sqrt{t}}$$

| | | |
|---|---|---|
| Price at Year 1 | 90.5 | |
| Duration now | 4.5 | |
| Duration at year 1 | 3.6 | |
| interest rate volatility | 30% | |
| Price volatility | 7.56% | |
| d1 , N(d1) | -0.5555319 | 0.28926545 |
| d2, N(d2) | -0.6311319 | 0.263977144 |
| C | 1.13859923 | |

Assumptions of black models applies.

The convexity term is taken to be small to apply the price and interest rate volatility approximation.

Days convention is ignored.

[7]

**[10 Marks]**

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