

INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

27th March 2021

Subject SP6 – Financial Derivatives

Time allowed: 3 Hours 30 Minutes (14.30 – 18.00 Hours)

Total Marks: 100

INSTRUCTIONS TO THE CANDIDATES

- 1. Please read the instructions to examinees sent along with hall ticket carefully and follow without exception.*
- 2. The answers are not expected to be any country or jurisdiction specific. However, if Examples/illustrations are required for any answer, the country or jurisdiction from which they are drawn should be mentioned.*
- 3. Mark allocations are shown in brackets.*

- Q. 1)** You are an investment actuary offering consulting services specializing in derivatives. Suppose it is October 1, 2015 today and a client comes to you for advice. He is holding 1000 shares of Actuarial Corporation of Utopia (ACU) and he has observed that the futures are trading at a substantial discount to the market. The client wants to sell the stock and buy the futures.

You observe the following (as of October 1, 2015):

- ACU spot price: Rs. 650 / Rs. 652
- ACU 1-month futures (i.e., expiring on October 31, 2015): Rs. 590 / Rs. 594
- Interest rate for all terms: 5% / 6%
- Expected dividend per share: Rs. 45
- Ex – Dividend Date: 15th October 2015
- Brokerage on Spot Transaction: 1% of the spot value
- Brokerage on Futures Transaction: 0.5% of the future price
- Margin required at any point in time: 5% of the underlying value as at the close of the day (assume Rs. 651)

Further, for this question assume that futures will NOT be settled daily and margin will be released at the maturity along with the settlement.

Based on the above information,

- i) What will be your advice with respect to the soundness of the client's strategy of selling the stock and buying futures? (9)
- ii) What are the risks that the client will be exposed to while implementing the strategy? (5)
- [14]

- Q. 2)** The IOA managed fund has Rs. 100m of assets. It has always invested 50% of its funds to track a total return equity index, holding the remaining 50% in cash.

Considering uncertain equity markets, it has been proposed that the fund protects its entire equity holding with a one-year put option on the tracked index with a strike 20% below the current index level. The purchase cost would be funded by the selling of a call option on the same index for the same notional, with a strike chosen to make the prices of the call and the put equal.

The risk-free rate is 2% per annum continuously compounded at all durations and the implied volatility of the put is 25% per annum.

- i) Calculate the price of the put option. (3)

It has been determined that the appropriate call option has a strike equal to 124% of the current index level. The implied volatility of the call is 20% and its price is equal to the put price.

IOA's risk department is content that the fund's mandate allows the use of such derivatives. However, it is questioning whether the fund managers have fully understood the impact of the transactions being proposed.

As a starting point, the risk department wishes to calculate the short-term sensitivity of the overall portfolio to equity prices (assuming that the put and call transactions take place) and to express this as an "effective equity exposure" comparable to the current 50%.

- ii) Calculate the fund's effective equity exposure. (5)
- iii) Explain how the effective equity exposure would change if:
- equity prices suddenly increase.
 - market implied volatilities increase.
 - the fund managers choose to use options with a shorter term.
 - the fund managers choose to use a lower strike for the put and a higher strike for the call. (4)

The risk department is also interested in the fund's sensitivity to interest rates, which it wants to express as an "effective bond duration". This duration is defined to be the term of a zero-coupon bond that, if it replaced the 50% cash in a fund with no derivatives, would result in the same overall interest rate sensitivity.

- iv) Using the rho of the European call and put option, calculate the effective bond duration of the fund after the derivative transactions. (5)
- v) Outline two other main changes to the risk profile of the fund that would arise as a result of the proposed derivatives strategy. (3)
- [20]

Q. 3) The Cox-Ingersoll-Ross (CIR) model is a one-factor interest-rate model of the form:

$$dr = \alpha(\mu - r)dt + \sigma\sqrt{r} dz$$

which leads to the solution at time t for the prices $B(t, T)$ of zero-coupon bonds of maturity T as follows:

$$B(t, T) = \exp [a(\tau) - b(\tau).r(t)]$$

Where,

$$a(\tau) = c_3 \ln \left\{ \frac{c_1 \exp(c_2 \tau)}{c_2 [\exp(c_1 \tau) - 1] + c_1} \right\} \text{ and } b(\tau) = \frac{\exp(c_1 \tau) - 1}{c_2 [\exp(c_1 \tau) - 1] + c_1},$$

$\tau = T - t$ and c_1, c_2 and c_3 are all positive constants ($c_1 > c_2$) containing terms in α, μ , and σ .

- i) Answer the following:

- a) List the key features that a good model of the entire interest rate yield curve should have.
- b) Discuss how well the CIR model matches the criteria.
- c) Describe in particular how the CIR model copes with two typical problems facing interest rate models: too wide a dispersion of rates over time due to future uncertainty and allowing interest rates to become negative.

(8)

- ii) By considering the behaviour of the logarithm of the bond price, prove that for large τ :

$$B(t, T) \approx \exp(-R \tau)$$

where $R = (c_1 - c_2) c_3$ is a constant long-term rate.

(5)

- iii) The CIR model is a no-arbitrage model. Outline what is meant by this term, and the significance of such a property for an interest-rate model.

(3)

[16]

Q. 4) XYZ Corp. is a financial institution that specialises in the interest rate derivatives. The company is planning to launch a Structured Fixed to floating swap – fixed rate receiver only accrues fixed rate when the interbank rates are between 6%-7% otherwise they accrue nothing.

- i) Describe structured products. What are the key requirements for a structured product? What are the key risks of a structured product? (6)
- ii) Propose a method to be used for valuing the swap structure by describing the cashflows and identifying the replicating financial instruments. State the assumptions and simplification required for the valuations. (3)
- iii) Describe the shape of the delta of the above structured product in relation to a normal call/put option. Describe the behavior of Gamma at the boundaries. (4)
- iv) XYZ has been approached by an insurance company and they want flexibility to specify the range instead of 6-7% without altering the width at every reset point. What could be the issues due to this on the above structure and what are the techniques used in valuing these structures. (3)

[16]

Q. 5) An equity brokerage firm allows its client to leverage to help them get more exposure than initial equity. They control the exposure by use of margin rate which is investors equity/stock value.

An Investor has put up 1cr to take a long position in a stock with a price of Rs 10000.

- i) Given initial margin of 80% calculate number of shares purchased by the investor and exposure of the brokerage firm. (2)

- ii) If the brokerage firm asks for margin call when margin rate falls below 50%, at what share price the company would have to pay the margin and what will be the investor equity at margin call. (4)
- iii) Calculate the return on stock and return on your equity at margin call price. Discuss the significance of the ratio of these returns. (3)
- iv) Calculate profit /loss percent for the investor in 1 year if the following things happen:
- Share price goes to 7,000.
 - Company declares the dividend of Rs. 350 per share.
 - Brokerage firm charges an interest of 10% per annum on the loan. (4)
- v) Another investor uses a capital of 1 Cr to short the equity with price Rs 10000 with margin of 50%. He expects the margin call at 30%. Calculate expected loss and return on equity at share price of margin rate. (5)
- [18]**

Q. 6) The market currently uses IBOR Interbank offer rate which is an estimate of benchmark interest rate at which large banks lend to one another.

- i) State the advantages and disadvantages/moral hazards of using IBOR. (3)
- ii) The regulator has decided to use secure overnight financing rate based on derivatives and loans instead of IBOR. Discuss market implication of this shift by considering liquidity, volatility and operational challenges. (3)
- [6]**

Q. 7) With interest environment becoming volatile, issuers are increasingly preferring callable bonds.

- i) Describe what are callable bonds. Describe forward price and yield volatility at call date and discuss the relation between them. (3)
- ii) Calculate the price of the option in the callable bond given the following information.
- Redemption of callable bond is at par minus coupon.
 - Maturity 5 years with call date 1 year.
 - Coupon for the bond is 5%.
 - Non -callable bond with same terms have continuously compounded yields of 7% and duration of 4.5 years. Assume each year represent 20% of the duration.
 - One year forward yield volatility is 30%.
 - Risk free rate is 5% per annum continuously compounded.

State any other simplification done for estimating the value of the option in the bond. (7)

[10]
