INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

22nd November 2019

Subject CM2A – Financial Engineering and Loss Reserving (Paper A)

Time allowed: 3 Hours 15 Minutes (14.45 – 18.00 Hours)

Total Marks: 100

INSTRUCTIONS TO THE CANDIDATES

1. ***Please read the instructions inside the cover page of answer booklet and instructions to examinees sent along with hall ticket carefully and follow without exception.***
2. ***Mark allocations are shown in brackets.***
3. ***Attempt all questions, beginning your answer to each question on a separate sheet.***
4. ***Please check if you have received complete Question Paper and no page is missing. If so, kindly get new set of Question Paper from the Invigilator.***

AT THE END OF THE EXAMINATION

Please return your answer book and this question paper to the supervisor separately. You are not allowed to carry the question paper in any form with you.

Q. 1) A market consists of shares of 3 companies *A*, *B* and *C* with capitalization of Rs 10,000 crores, Rs 15,000 crores and Rs 10,000 crores respectively.

Annual returns on the three shares (*RA*, *RB* and *RC*) have the following characteristics:

Company	*Standard deviation*
A	30%
B	20%
C	10%

The expected rate of return on the market portfolio is 12% p.a. The correlation between the returns on each pair of shares is 0.5.

The risk-free rate of return is 7% p.a.

i) Find the expected returns on *A*, *B* and *C* if the CAPM is assumed to hold. (9)

ii) Split the variances of returns into systematic and specific risk for every company using the values in the question & the values obtained in (i) above. (4)

[13]

Q. 2) **i)** State Lundberg's inequality, with proper explanation of all the variables/parameters. (2)

ii) A Poisson claims process has a security loading of $\theta = 1/3$ and claim size density function, $f(x) = 2e^{-2x}$

a) Derive the Moment Generating Function (MGF) for claim size distribution, and state the values for which it is valid. (3)

b) Calculate the value of adjustment coefficient. (4)

[9]

Q. 3) The process X has the stochastic differential equation, $dX_t = \alpha\,\mu\,(T-t)\; dt + \sigma\sqrt{(T-t)}\; dZ_t$, *where* $\alpha > 0$ & μ are fixed parameters and *Z* is a standard Brownian motion under Q.

The function *f* is given by $f(x, t) = e^{(m(T-t)-x)}$ where m is a differentiable function.

Find $\frac{\partial m}{\partial t}$ if $f(x, t)$ is a martingale. **[6]**

Q. 4) Two bookmakers, Luckworth and Dewis, offer the following fair gambles:

Luckworth offers Rs 100 for correctly predicting the outcome of tossing a fair coin on an initial payment of Rs L.

Dewis offers Rs 100s where s is generated by a uniform random number generator from the interval [0,1] on an initial payment of Rs D.

i) For both the gambling options, state the returns from a buyer's (i.e. gambler's) perspective and find the values of L and D assuming the gambles to be fair. (2)

ii) For returns from both gambling options from a buyer's perspective, compute the following risk measures:

a. Variance
b. Downside Semi Variance
c. 90% Value at risk
d. Expected shortfall (at -Rs 10 threshold) (10)

iii) Comment on the results of the risk measures computed in the previous sub-part. (2)

iv) If Frank goes for Luckworth's offer, Tony opts for Dewis's, and Steven chooses neither, what can you say about their individual risk preferences? How do they compare? (4)

[18]

Q. 5) In the country of Derivativa, apart from the usual European and American call and put options, a variety of exotic options are traded. Among them are digital call, digital put and digital range options which have the following meanings:

Digital call option: The writer / seller pays Re 1 to the holder / buyer if the stock price at expiry is more or equal to than a pre-specified strike price.

Digital put option: The writer / seller pays Re 1 to the holder / buyer if the stock price at expiry is less than a pre-specified strike price.

Digital range option: The writer / seller pays Re 1 to the holder / buyer if the stock price at expiry is less than the upper strike but not less than the lower strike (i.e. between the two pre-specified strike prices).

Further, stock prices in Derivativa are assumed to follow geometric Brownian motion and the constant continuously compounded risk-free rate of interest is r.

i) What does the following equation mean? Explain each term.
$V_t=e^{-r(T-t)}E_Q[X_T|F_t]$ (2)

ii) Using the above equation, derive the formula for the price of a digital call option with strike price K and expiring after time T on a non-dividend paying stock currently valued at S_0. (6)

iii) Using the formula for the price of a digital call option derived above, derive a formula for the price of a digital range option with lower strike K_L and upper strike K_U. (Assume $K_L < K_U$.) (2)

iv) Similar to the put-call parity relationship, derive a relationship between the prices of a digital call and a digital put option with the same strike price and expiry. (2)

v) The above pricing derivations were performed by the martingale approach. What are the advantages of the martingale approach compared to the PDE approach? (3)

vi) Despite the above advantages, why do we still sometimes use the PDE approach? (2)

[17]

Q. 6) A Bond investor does not know the Price to pay to a Zero-coupon Bond of a new company 'A' that has come for issuance. The term to expiry is 7 years.

However, a similar Company 'B' that had issued a Zero-coupon bond is trading now. Total number of Bonds that are being traded is 10 lakhs.

The financial details of Company 'B' are given below:

The current value of liability valued using risk-free rate is Rs 15 crore with maturity of 5 years. The value of the assets of the company is Rs 20 crore with an annual volatility of 40%. The company is expected to be wound up after 5 years when the assets will be used to pay off the bond holders with the remainder being distributed to the equity holders. A constant risk-free rate of return is 7% p.a. (compounded continuously).

Find the Price for 100 nominal Bond that the Investor may be happy to pay if he puts a constant credit spread of 100 basis points p.a. higher than that of spread of bonds of Company B. **[8]**

Q. 7) **i)** What is iso-elastic utility function? (1)

ii) Prove that the function $U(x) = (x^{5\alpha} - 1)/10\alpha$ is an iso-elastic utility function. (4)

iii) An individual is considering purchasing an insurance policy to protect against the random loss of Rs 500 that occur with the probability of 0.5. The utility function of the individual is given by $U(x) = \text{Log}(x)$.

The initial wealth of the individual is Rs 1000. Calculate the insurance premium for the individual. (4)

Note: Please consider the exponential base of the functional Log, for all calculations.

[9]

Q. 8) A general insurance company is writing motor business for the last 5 years. The single premium charged for the policy in a particular year is INR 15,000 and the amount of claim reported in that year is INR 3000. There are 4 months left to expire the policy and the tenure of the policy is 12 months.

i) Calculate the earned premium and the Loss ratio of the insurer, ignoring any IBNR claims. (2)

ii) The claim payments of the insurer for different years is given below:

		Development Year				
		1	2	3	4	5
Accident Year	1	25,000	15,000	10,000	8,000	7,800
	2	27,000	23,000	20,000	19,000	-
	3	29,000	26,000	23,400	-	-
	4	31,500	28,900	-	-	-
	5	33,000	-	-	-	-

The earned premium for each Accident year is 70,000 for year 1; 83,000 for year 2; 91,000 for year 3; 102,000 for year 4 and 110,000 for year 5.

Apply the Bornhuetter – Ferguson method to estimate the amount of claims yet to be paid, stating any assumptions that you make. (8)

[10]

Q. 9) **i)** Explain arbitrage opportunity. (2)

ii) Define law of one price. (2)

iii) Let p_t be the price of 3-month European put option on a share with current share price Rs 125 and strike price of Rs 120. The call options on the same share are priced at Rs 30 and the current risk- free force of interest is r= 5% p.a.

a) If dividends are payable continuously at a rate of q= 15% p.a., calculate p_t. (2)

b) Explain the strategy for arbitrage profit if, instead, the price of the put option is Rs 23. (4)

[10]

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