

**INSTITUTE OF ACTUARIES OF INDIA**

**EXAMINATIONS**

**27<sup>th</sup> November 2020**

**Subject CM1A – Actuarial Mathematics (Paper A)**

**Time allowed: 3 Hours 30 Minutes (09.30 – 13.00 Hours)**

**Total Marks: 100**

**Q. 1) i)** Explain why a stochastic model would be better than a deterministic model for modelling the guarantees. (3)

**ii)** Explain how a deterministic model might be used to validate model outcomes where a stochastic approach has been selected. (2)  
[5]

**Q. 2)** Describe why a company might wish to use sensitivity analysis as part of its profit testing exercise. [4]

**Q. 3)** A customer is repaying a loan with payments of 1 at the end of each year for  $n$  years. The annual effective interest rate on the loan is  $i$ . The amount of interest paid in year  $t$  plus the amount of principal repaid in year  $t + 1$  equals  $X$ . Determine which of the following is equal to  $X$ .

a)  $1 + \frac{v^{n-t}}{d}$

b)  $1 + v^{n-t}i$

c)  $1 + v^{n-t}d$

d)  $1 + v^{n-t}$

[3]

**Q. 4)** A perpetuity costs 79.56 and makes end-of-year payments. The perpetuity pays 1 at the end of year 2, 2 at the end of year 3, ...,  $n$  at the end of year  $(n+1)$ . After year  $(n+1)$ , the payments remain constant at  $n$ . The annual effective interest rate is 10.5%.

Please select the correct value of  $n$  from the options provided below.

a) 17

b) 19

c) 21

d) 23

[4]

**Q. 5) i)** You are provided with the ultimate and select mortality rates below. An insurance provider sells a policy with a term of 2 years providing death benefit of INR 100,000 at the end of the year in which the policyholder dies to two customers A & B both aged 37 years exact. The insurer undertakes medical underwriting in respect of A but not in respect of B. The term of both policies is 2 years. Assuming a discount rate of 6%, what is the excess of the expected cost of claim for B over A (rounded to the nearest integer)?

Age (x)	$[x]$	$[x-1]+1$	$[x-2]+2$
37	0.000878	0.000969	0.001072
38	0.000918	0.001018	0.001136
39	0.000965	0.001079	0.001213

Please select the correct answer from the options provided below:

a) 9

b) 29

c) 33

d) 37

(4)

**ii)** Briefly explain the result (1)

[5]

- Q. 6)** A company is selling non-participating endowment product with a policy term of 10 years and premium payable as single premium only.

For each of the below, briefly comment on whether it would be considered in the calculation of office premium. Also state briefly how each of the below will impact the premium calculation.

- a. Remuneration of the Chief Executive Officer of the insurance company
- b. Premium being charged by another insurance company on a similar insurance product
- c. Expectation of future inflation levels
- d. Expectation of yields on fixed interest bonds with a term of 10 years

[4]

- Q. 7)** A fixed interest bond with an outstanding term of 3 years is redeemable at INR 94 per 100 nominal. The bond pays a coupon of 8% per annum payable semi-annually. Calculate the yield to maturity for this bond.

Select the correct answer from the options provided below.

- a) 9.35%
- b) 10.45%
- c) 10.65%
- d) 11.55%

[4]

- Q. 8)** Consider a with-profit insurance policy with a sum assured of INR 500,000. At the end of each policy year, the company declares a bonus to the policyholder. The sum assured along with all accrued bonuses is payable on completion of the policy term of 10 years. Rank the accrued bonus at the end of 4 years under the bonus structures listed below:

- A. 4.5% simple bonus for first 5 policy years and 5.5% simple bonus thereafter
- B. 4% compound bonus rate each year
- C. Super compound bonus at 4% on sum assured and 5% on bonuses added in the past.

Select the correct option from the below:

- a)  $A > B > C$
- b)  $A > C > B$
- c)  $C > B > A$
- d)  $C > A > B$

[4]

- Q. 9)** i) Life insurance company sells 1,000 whole life annuities on 1 April 2019 to policyholders aged 65 exact. Each annuity is for INR 250,000 payable annually in arrears. 5 annuitants die during the financial year. The Company holds reserves using the following basis:

- Mortality PFA92C202
- Interest 4% per annum

Calculate the profit or loss from mortality for this group for the financial year ending 31 March 2020.

(4)

- ii) The staff of a company are subject to two modes of decrement; death and withdrawal from employment. Decrement due to death take place uniformly over the age in the associated single decrement table; 50% of the decrements due to withdrawal occur uniformly over the year of age and the balance occurs at the end of the year of age, in the associated single-decrement table.

You are given the independent rate of mortality as 0.001 per year of age and independent rate of withdrawal as 0.1 per year of age.

Calculate the probability that a new employee aged exactly 20 will die as an employee at age 21 last birthday.

(5)  
[9]

- Q. 10)** iCare Life offers a product to its customers providing benefits on death as well as survival. The product has a defined “basic sum assured”. Upon death of the policyholder during the policy term, the product pays death benefit equal to twice the basic sum assured immediately upon death. Upon survival to the end of the policy term, the policy provides maturity benefit equal to 110% of the basic sum assured. Further, upon completion of the premium payment term, the policy pays a benefit of 5% of sum basic sum assured at the end of the year provided the policyholder is still alive.

The pricing basis for the product is:

- Mortality: AM92 Ultimate table.
- Initial expense: 15% of annual premium amount
- Variable renewal expenses are 2% of annual premium income from second policy year onwards, incurred over the premium payment term.
- Fixed annual renewal expenses are INR 500 incurred from second policy year onwards over the policy term. These are incurred at the end of the year. Expense inflation assumed is 0%.
- Initial commission is 10% of annual premium; and renewal commission is 5% of annual premium.
- Assume no other costs / benefits.  
Interest rate is 6% p.a.

The company also includes a profit loading of 10% of annual premium in its pricing basis.

Other key features of this product are:

- It has a premium payment term of 5 years and premiums are payable annually at the start of each policy year.
  - It has a policy term of 10 years.
- i) For a policyholder aged 30 years exact, determine the present value of benefits payable under this product where the basic sum assured is INR 100,000. Select from the options provided below, rounded to the nearest thousand (i.e. if the answer is 5,650 then nearest thousand is 6000):
- a) 65,000
  - b) 66,000
  - c) 60,000
  - d) 62,000

(4)

ii) Calculate the annual premium consistent with the pricing basis above (rounded to the nearest integer) (10)

iii) The insurance company is now looking to launch a new variant of this product under which the policyholder will be given an option to receive the maturity benefit upon completion of the policy term in staggered equal amounts (called “income benefit”) over a period of 10 years (called “income period”) starting immediately upon completion of the policy term. Payments during the income period are guaranteed in nature. The policyholder can opt for this option at the end of the policy term. The annual guaranteed payments are defined as 12.5% of maturity benefit (defined above). Determine the interest rate being guaranteed to the policyholder upon conversion to the income payments and explain whether this will result in an increase or decrease in premiums? (4)

[18]

**Q. 11)** You are working in a life insurance company, Sanits, as a part of actuarial team. Due to sudden outbreak of pandemic, the entire state is in lockdown mode and employees are advised to work from home. The infected cases are growing manifold with each day and the lockdown is expected to continue for 3 months.

The number of infected policyholders increases with the transition intensities,  $\delta(t)$  which is a function of time and at any time  $t$  (measured in months) is given by:

$\delta(t) =$	0.10	for $t \leq 1$
	$0.8 + 0.05t$	for $t > 1$ and $t \leq 2$
	$0.05 + 0.06t$	for $t > 2$ and $t \leq 3$

Assume that the number of infected policyholders at time 0 is 0.5 per 1000 and the total number of policyholders exposed are 10,00,000.

i) Calculate likely number of affected policies at the end of each month to the nearest whole number. (6)

Based on the historical trend it has been observed that the death rate is 1.00% per month. The number of deaths per month is calculated on the average number of infected policyholders during the month. Claim payments are being made at the end of the month.

Assume that sum assured for each policy is INR 50,000.

ii) Calculate likely number of deaths to the nearest whole number and claim amounts respectively at the end of each month. (6)

As per the Business Continuity Plan, company is considering to go with complete digitisation for 3 months. The digitisation exercise is expected to incur the following costs:

- Initial Cost of purchasing 5 laptops– Rs 35,000/- each
- Initial Cost of purchasing 20 datacard– Rs 400/- each
- Monthly rental for each datacard for 3 months payable at the end of month– Rs 500/- per month
- Monthly maintenance expense for digitisation of Rs 20,000/- for 3 months incurred at the end of the month

- Fumigation cost per month is Rs 5,000/- for 3 months and is incurred at the start of each month
- Expense on security features installation at the outset – Rs 2,00,000/-

The Company, Sanits, is currently experiencing capital issues and decides to borrow capital from its group company, Locks, of Rs 10,00,00,000/-, which it would earmark as a working capital to deal with this pandemic. The group company Locks, is operating in different market where interest rate is low. Sanits, seeing the arbitrage opportunity, borrows the capital as an interest only loan with capital payable at the end of three months. The rate of interest applicable on borrowed capital is 6% p.a. convertible monthly. The company Sanits intends to invest the amount in the local market in instruments providing 12% p.a. convertible monthly.

All expenses, including claims (calculated above) have to be met from the invested working capital. Calculate:

- iii) Amount of surplus available at the end of 3 months after meeting all expenses, likely claims and capital repayment with interest. Comment whether this arrangement is viable. (10)
- iv) If the arrangement is not viable, what is the likely capital the company, Sanits, should seek from its Group Company, Locks. (2)

Assume interest rate of 12% p.a. convertible monthly for discounting purposes and that both companies operate in the same currency.

[24]

- Q. 12)** A life insurance company issues a 4-year unit-linked endowment assurance contract to a male life aged 40 exact under which level premiums of INR 100,000 per annum are payable in advance. In the first year, 90% of the premium is allocated to units and 101.5% in the second and subsequent years. The units are subject to a bid-offer spread of 5% and an annual management charge of 0.5% of the bid value of the units is deducted at the end of each year.

If the policyholder dies during the term of the policy, the death benefit of INR 500,000 or the bid value of the units after the deduction of the management charge, whichever is higher, is payable at the end of the year of death. On surrender or on survival to the end of the term, the bid value of the units is payable at the end of the year of exit. The company uses the following assumptions in its profit test of this contract:

- Rate of growth on assets in the unit fund: 6% per annum
- Rate of interest on non-unit fund cashflows: 4% per annum
- Independent rates of mortality AM92 Select
- Independent rate of withdrawal: 10% per annum in the first policy year;  
5% per annum in the second and subsequent policy years.
- Initial expenses INR 2500
- Renewal expenses INR 500 per annum on second & subsequent premium dates
- Initial commission 10% of first premium
- Renewal commission 4.5% of the second and subsequent years premiums

- Risk discount rate 8% per annum
- Expenses and Commission, both initial and regular, occur at the same time as the payment of premium. Decrements are uniformly distributed over the year.

All cash flows are rounded to the nearest rupee.

i) Calculate and select the correct value of projected Unit Fund at the end of year 1, year 2, year 3 and year 4 from the following:

- a) 90,177; 196,809; 309,274 and 427,891
- b) 91,878; 202,350; 321,064 and 448,634
- c) 88,475; 191,335; 297,774 and 407,917
- d) 90,177; 196,809; 309,274 and 423,854

(4)

ii) Calculate and select the correct value of projected net cash flows at the end of each year of year 1, year 2, year 3 and year 4 from the following:

- a) 2,226; -777; -133 and 583
- b) 2,227; -772; -121 and 607
- c) 2,210; -785; -139 and 581
- d) 2,211; -779; -126 and 606

(5)

iii) Calculate the profit margin on the assumption that the office does not zeroise future negative cash flows.

(3)

Suppose the office does zeroise future negative cashflows, answer the following.

iv) Calculate and select the expected provisions that must be set up at the end of each year, per policy in force at the start of each year from the following:

- a) 776.83; 121.37; 0
- b) 763.03; 110.42; 0
- c) 788.48; 126.85; 0
- d) 773.03; 114.99; 0

(2)

v) Calculate and select the expected present value of profit allowing for the cost of setting up these provisions.

- a) 1688.90
- b) 1716.89
- c) 1662.11
- d) 1692.22

(2)

**[16]**

\*\*\*\*\*