Institute of Actuaries of India

Subject CM1A – Actuarial Mathematics (Paper A)

November 2020 Examination

INDICATIVE SOLUTION

Introduction

The indicative solution has been written by the Examiners with the aim of helping candidates. The solutions given are only indicative. It is realized that there could be other points as valid answers and examiner have given credit for any alternative approach or interpretation which they consider to be reasonable.

Solution 1:

i) A stochastic model runs many different investment scenarios, where the future investment returns are governed by a probability distribution function. [0.5]
Some of those investment scenarios will show poor investment performance, where the fund value falls below the original premium level. [0.5]
In these scenarios the cost of the money back guarantee will result in a negative cash flow to the company. [0.5]
The company would run many simulations and the cost of the guarantee determined by the model will be the average shortfall over all simulations. [0.5]
As a result, the company will recognize the need to charge a higher annual management charge for offering the guarantee. [0.5]
The stochastic model is clearly superior in that it can correctly determine the need for such a charge. [0.5]
[3]

ii) A deterministic result on best estimate assumptions could be compared with the mean and median outcomes from a stochastic approach.
A deterministic model may also be used to calculate the expected or median outcome, with a stochastic approach being used to estimate the volatility around the central outcome.
[2]
[5 Marks]

Solution 2:

Sensitivity analysis may be carried out at an individual policy level and at a portfolio level.
At an individual policy level, sensitivity analysis allows the company to understand the impact of mis-estimating parameter values in the model.
It can help show what the reductions would be in profits emerging, return on capital or other metrics targeted.
This may help assess what margins may be included in the parameter values for the risk that are not borne out in reality.
At a portfolio level, sensitivity analysis can be used to assess the impact of shifts in mix of business.
Some parts of the portfolio may be more profitable than others and this analysis will highlight the possible impact on overall profitability of the product.
Sensitivity analysis on the volume of business can also be used to assess the overall profits emerging.
This may be useful to validate the viability of any development expenditure associated that may be associated with the pricing exercise.
This can also help the company to understand the possible risk to its capital position if volumes are more than expected.
[4 Marks]

Solution 3:

Answer : (c)
Year t interest is i ( )] = 1- ( ) =1-
Year (t+1) principal repaid is 1- (1-) =
X= 1- +
= 1+ (1-v)
= 1+ d
[3 Marks]

Solution 4:

Answer : c)
79.56 = v( )] +——

$= v \frac{1}{} + \frac{}{}$

$= v \frac{1}{} - \frac{}{} + \frac{}{}$

$= v \frac{1}{}$

$= \frac{1}{}$

$= \frac{}{}$

$= \frac{}{(\ .\ )}$

$= \frac{}{.}$

$\Rightarrow 1 - = .871145$

$\Rightarrow = .12285$

$\Rightarrow$

$1.105 \quad = .12285$

$\Rightarrow = - \frac{(\ .\ )}{(\ .\ )}$

$\Rightarrow = 21$

[4 Marks]

Solution 5:

i) **We will use Select Mortality rates for Policyholder A and Ultimate Mortality rates for Policyholder B to reflect levels of underwriting.**
Expected cost of claims for customer A
$= 100{,}000 * (q_{[37]} / 1.06 + (1-q_{[37]}) * (q_{[38-1]+1}) / 1.06^{\wedge 2})$
$= 100{,}000 * (0.000878/1.06 + (1-0.000878)*(0.001018)/1.06^{\wedge 2})$
$= 173.3523$

Expected cost of claims for customer B
$= 100{,}000 * (q_{[37-2]+2} / 1.06 + (1-q_{[37-2]+2}) * (q_{[38-2]+2}) / 1.06^{\wedge 2})$
$= 100{,}000 * (0.001072/1.06 + (1-0.001072)*(0.001136)/1.06^{\wedge 2})$
$= 202.1273$
Difference = 202.12 – 173.3 = 28.8 = 29 (rounded to nearest integer)

[4]

ii) **The expected cost of claim is lower for customer A as he has been subject of medical underwriting. This is as expected as purpose of underwriting is to screen out bad risks and company is more aware of state of health of customer A.**

[1]
[5 Marks]

Solution 6:

a. **Yes, as this would form part of indirect expenses of the Company and would be included in the expense assumption.**

b. **Yes, but only for competitive reasons to ensure premium is not out of line with the market. This will therefore act as a reasonableness check on the premium rate.**

c. **Yes, to allow for expected inflation on current costs of the company over the 10 year term of the policy.**

d. **Yes, to the extent this affects the choice of interest rate for discounting cash flows.**

[4 Marks]

Solution 7:

Answer : (c)

Let yield to maturity be = i per annum.
Then, $94 = 4(v^{0.5} + v + v^{1.5} + v^2 + v^{2.5)} + 104v^3$; where $v = 1/(1+i)$
At i = 10%, RHS = 95.51
At i = 11%, RHS = 93.19
So, interpolating i ~ 10% + (95.51 – 94)* 1% / (95.51 – 93.19) = 10.65%

[4 Marks]

Solution 8:

Answer : (b)

A. Simple bonus at the end of 4 years = 4.5% * 500,000 * 4 = 90,000

B. Compound bonus = 500,000 (1+4%)^4 – 500,000 = 84,929

C. Super compound bonus at 4% of SA and 5% of bonus = 86,203

End of year	Bonus on sum assured	Bonus on past bonus	Total accrued bonus
1	20,000	0	20,000
2	20,000	=20,000 * 5% = 1,000	=20,000 + 20,000 + 1,000 = 41,000
3	20,000	= 41,000 * 5% = 2050	63,050
4	20,000	= 63,050*5% = 3153	86,203

So, A > C > B

[4 Marks]

Solution 9:

i) The Death Strain at risk per policy is:

[0 − (payment due 31.03.2020 + reserve @ 31.03.2020] = -250,000 * PV of annuity at 31.03.2020 [1]

Expected DS = -q65 *1,000*250,000*PV of annuity at 31.03.2020
= −(0.004681)(250,000,000)(14.494) = −16,961,600 [1]

Actual DS = −5*250,000* PV of annuity at 31.03.2020 = −18,117,500 [1]
Profit = EDS − ADS = -16,961,600 + 18,117,500 = 1,155,900 profit [1]
[4]

ii) Construct a multiple decrement table.

Age	No. alive	No. deaths	No. Withdrawals over the year	No. Withdrawals at the year end
20	100,000	97.50	4997.5	4745.25
21	90,159.75	87.90		

At age 20, no. of deaths = 100000*0.001(1-0.5*0.05) = 97.50 [1]
no. of withdrawals over year = 100000*0.05*(1-0.5*.001) = 4997.5 [1]
no. of withdrawals at year end = 100000*(1-0.05)*(1-0.001)* 0.05 = 4745.25 [1]

At age 21, no. of deaths = 90159.75*0.001(1-0.5*0.05) = 87.90 [1]
Required probability = 87.90/100000 = 0.000879 [1]
[5]

[9 Marks]

Solution 10

i) Answer : b)

Present value of benefits:
= Immediate payment on death + benefit on maturity + benefit on completion of premium term

$= 200{,}000\ A^{1}_{30:10}$ (continuous) + 110% * 100,000 $A^{1}_{30:10}$ + 5% * 100,000 * $A^{1}_{30:5}$ Now,

$A^{1}_{30:10}$ (continuous) = $(1+6\%)^{1/2} * A^{1}_{30:10}$; where

$A^{1}_{30:10} = A_{30} - v^{10}\ l_{40} / l_{30}\ A_{40}$ = 0.07328 – 1.06^(-10) * 9856.2863/9925.2094 *0.12313

= 0.005002304

So, $A^{1}_{30:10}$ (continuous) = 0.005150187

$A^{1}_{30:10} = l_{40} / l_{30} * 1.06^{(-10)}$ = 9856.2863/9925.2094 * 1.06^(-10)

= 0.554517

$A^{1}_{30:5} = l_{35} / l_{30} * 1.06^{(-5)}$ = 9894.4299/9925.2094 * 1.06^(-5) =

0.7449408 Based on the above, present value of benefits =

= 200,000 * 0.005150187 + 110% * 100,000 * 0.554517 + 5% * 100,000 * 0.7449408

= 65,752 [4]

ii) Let the annual premium = P

Present value of premium = present value of benefits + present value of expenses + present value of commissions + profit loading [1]

Present value of premiums = P $\ddot{a}_{30:5}$;

Where $\ddot{a}_{30:5} = \ddot{a}_{30} - v^{5} * {}_{5}p_{30} * \ddot{a}_{35}$ = 16.372 – 1.06^(-5) * 9894.4299/ 9925.2094 * 15.990

= 4.460396

Present value of premiums = P x 4.460396; [2]

Present value of benefits = as calculated in part a = 65,752

Present value of expenses = 15% * P + (2% * P * $a_{30:4}$) + 500 * $a_{31:9}$ * ${}_{1}p_{30}$ * v [1]

Where $a_{30:4} = a_{30} - v^{4} * {}_{4}p_{30} * a_{34} = (\ddot{a}_{30} - 1) - v^{4} * l_{34} / l_{30} * (\ddot{a}_{34} - 1)$

= (16.372-1) – 1.06^(-4) * 9900.9645/9925.2094 *

(16.075-1) =3.460357 [1]

And,

Where $a_{31:9} = a_{31} - v^{9} * {}_{9}p_{31} * a_{40} = (\ddot{a}_{31} - 1) - v^{9} * l_{40} / l_{31} * (\ddot{a}_{40} - 1)$

= (16.304-1) – 1.06^(-9) * 9856.2863/9919.3535 * (15.491-1)

= 6.78133 [1]

And,

1p30 = 1-q30 = 1 – 0.000590 = 0.99941 [0.5]

So, present value of expenses =

= 15% * P + 2% * P * 3.460357 + 500 * 6.78133 * 0.99941 / 1.06

= 0.219207 P + 3196.85 [1]

Present value of commissions = 10% * P + (5% * P) * $a_{30:4}$ = P * (0.1 + 0.05 * 3.460357) = 0.273 * P [1]

Profit loading = 10% * P [0.5]

So, as per principle of equivalence, we have:

4.460396 P = 65,752 + 0.219207 P + 3196.85 + 0.273 * P + 10% * P Therefore, P = 17825 [1]

[10]

iii) Present value of maturity benefit will be revised to:

= 100,000 * 110% * 12.5% * $\ddot{a}_{10}$ * $A_{30:10}^{1}$ [0.5]
Where $\ddot{a}_{10}$ = 7.3601* 1.06 = 7.8017
And $A_{30:10}^{1}$ = 0.554517 (calculated in part i)
So, PV of maturity benefit = 100000 * 110% * 12.5% * 7.8017 * 0.554517 [1]
= 59,485 From part i),
PV of maturity benefit in original policy = 110% * 100000 * 0.554517 = 60,996
Since there is a reduction in value of maturity value, there will be a reduction in premium amount as well. [0.5]

To determine effective interest rate, we need to solve for i in equation:
110,000 = 12.5% * 110,000 * $\ddot{a}_{10}$ [0.5]
8= $\ddot{a}_{10}$
8 = a9 + 1
7 = (1 –v9) / i
RHS = 7.10 when i = 5% and is 6.9522 when i = 5.5%. Interpolating, we get that i = 5.3% So, interest rate guaranteed to policyholder is 5.3% [1.5]
[4]
[18 Marks]

Solution 11:

i) Total Population = 10,00,000
Number of infected cases at t=0: 10,00,000 *0.5 / 1000 = 500 Number of infected cases at t =1: [0.5]
= 500* ∫(.) [0.5]
= 500*[.] [0.5]
= 500 *1.1052
= 553 rounded to nearest integer [0.5]

Number of infected cases at t =2:
=553* ∫ (. .) [0.5]
=553* [.] [0.5]
= 553 *2.3989 [0.5]
= 1327 rounded to nearest integer [0.5]

Number of infected cases at t =3:
=1327* ∫(. .) [0.5]
=1327* [.] [0.5]
=1327 *1.2214 [0.5]
= 1621 rounded to nearest integer [0.5]
[6]

ii)
Death rate = 1% of the average number of infected policyholders during the month.
Number of deaths and claim payments in 1st month:
Average population for first month = (500 + 553)/2 = 527 [0.5]
Deaths in first month = 527 *1% = 5 to nearest integer [0.5]
Claim amount payable = 50,000 * 5 = 2,50,000 [1]

Number of deaths and claim payments in 2nd month:
Average population for second month = (553 + 1327)/2 = 940 [0.5]
Deaths in second month = 940 *1% = 9 to nearest integer [0.5]

Claim amount payable = 50,000 * 9 = 4,50,000 **[1]**

Number of deaths and claim payments in 3rd month:
Average population for third month = (1327+1621)/2 = 1474 **[0.5]**
Deaths in third month = 1474 *1% = 15 to nearest integer **[0.5]**
Claim amount payable = 50,000 * 15 = 7,50,000 **[1]**
[6]

iii)

Initial expenses:
Cost of purchasing laptops:= 5* 35,000 = 1,75,000 **[0.5]**
Cost of purchasing data card: 20 * 400 = 8,000 **[0.5]**
Cost of security features installation : 2,00,000
Accumulated initial expense at the end of 3 months
$i_m = i_m^{(12)} / 12$ = 1% per month, where $i_m^{(12)}$ = 12% **[0.5]**
= (1,75,000 + 8,000 + 2,00,000)*1.01^3 **[0.5]**
= 3,94,605 **[0.5]**

Recurring expenses:
Accumulated rent of data card :
= 500 *{ 1.01^2+ 1.01^1+ 1} * 20 **[0.5]**
= 30,301 **[0.5]**

Accumulated maintenance expense of digitisation:
= 20,000 *{ 1.01^2+ 1.01^1+ 1} **[0.5]**
= 60,602 **[0.5]**

Accumulated fumigation expense:
= 5000 *{ 1.01^3+ 1.01^2+ 1.01} **[0.5]**
= 15,302 **[0.5]**

Total accumulated expenses, both initial and recurring
= 3,94,605 + 30,301 + 60,602 + 15,302
= 5,00,810 **[0.5]**

Total accumulated claims at the end of 3 months:
= 2,50,000 * 1.01^2 + 4,50,000 * 1.01 + 7,50,000 **[0.5]**
= 14,59,525 **[0.5]**

Capital repayment with Interest charged on capital at the end of 3 months:
= 10,00,00,000 * (1+ 3 * i_c) where $i_c = i_c^{(12)}/12$ = 0.06/12 **[0.5]**
= 10,15,00,000 **[0.5]**

Accumulated capital at end:
= 10,00,00,000 * (1.01)^3
= 10,30,30,100 **[0.5]**

Fund available at end of 3 months after capital repayment with interest:
= 10,30,30,100 - 10,15,00,000
= 15,30,100 **[0.5]**

Total expenses plus claims:
= Total accumulated Expenses + Total accumulated claims
= 5,00,810 + 14,59,525 **[0.5]**

Surplus at the end of 3 months:
= 15,30,100 - 19,60,335
= - 4,30,235
Hence, the arrangement is not viable and additional fund need to be borrowed. **[0.5]**
[10]

iv)

Likely capital to be borrowed:
Interest earned on capital = (1.01^3-1) = 0.030301 **[0.5]**
Interest paid on capital = 0.06/12 * 3 = 0.015 **[0.5]**
Capital to be borrowed
= Total expenses including claims / (Interest earned – Interest paid)
= 19,60,335 / (0.030301 – 0.015)
= 12,81,18,100 to the nearest hundred. **[1]**
[2]
[24 Marks]

Solution 12:

Annual premium 100,000.00
Risk discount rate 8.0%
Allocation % (1st yr) 90.0%
Allocation % (2nd yr +) 101.50%
Interest on Unit investments 6.0%
Interest on non-unit reserves 4.0%
Man charge 0.50%
B/O spread 5.0%
Minimum death benefit 5,00,000

	INR	%prm	Total
Initial expense /commission	2500	20.0%	12500
Renewal expense/commission	500	4.5%	5000

Multiple decrement table:

X	q^d_x	q^s_x		
40	0.000788	0.10		
41	0.000962	0.05		
42	0.001104	0.05		
43	0.001208	0.05		
x	$(aq)^d_x$	$(aq)^s_x$	(ap)	t-1(ap)
40	0.000749	0.09996	0.899291	1.000000
41	0.000938	0.04998	0.949086	0.899291
42	0.001076	0.04997	0.948951	0.853504
43	0.001178	0.04997	0.948852	0.809934

i)

Answer : a)

Unit fund (per policy at start of year)

	yr 1	yr 2	yr 3	yr 4
value of units at start of year	0	90,177	196809	309274
alloc	90,000	101,500	101,500	101,500
B/O	4,500	5,075	5075	5,075
interest	5,130	11,196	17,594	24,342
management charge	453	989	1,554	2,150
value of units at year end	90,177	196,809	309,274	427,891

[4]

ii)

Answer : a)

Cash flows (per policy at start of year)

	yr 1	yr 2	yr 3	yr 4
unallocated premium	10,000	-1,500.00	-1,500	-1500.00
B/O spread	4,500	5,075.00	5,075	5075.00
Expenses and commission	12,500	5,000.00	5,000	5000.00
interest	80	-57.00	-57.00	-57.00
man charge	453	989	1,554	2150.20
extra death benefit	307	284	205	85
end of year cashflow	2226	-777	-133	583

[5]

iii)

probability in force	1	0.899291	0.853504	0.809934
discount factor	0.925925	0.857338	0.793832	0.735029
expected p.v. of profit	1718.98			
Premium signature	100000	83267.69	73174.25	64295.17
Expected p.v. of premiums	320,737.09			
Profit Margin	0.54%			

[3]

iv)

Answer: a)

To calculate the expected provisions at the end of each year we have (utilising the end of year cashflow figures and decrement tables in (i) above):

$_2V=133/1.04=127.88$

Cashflow is year 2 = $-777-(ap)_{41^*}*{}_2V$

=>Cashflow is year 2 = $-777-(ap)_{41^*}*127.88 = -898.37$

=> $_1V = 898.37/1.04$

=> $_1V = 863.82$

These need to be adjusted as the question asks for the values in respect of the beginning of the year. Thus we have:

Year 2 127.88(ap)41 = 121.37

Year 1 863.82(ap)40 = 776.83

[2]

v)

Answer: a)

Based on the expected provisions calculated in (a) above, the cash flow for years 2 and will be zeroised whilst year 1 will become:

2226 -776.83 = 1,449.17

Hence the table blow can now be completed for the revised profit margin revised end of year cash flow:

	1,449.17	0	0	583
probability in force	1	0.899291	0.853504	0.809934
discount factor	0.925925	0.85733	0.793832	0.735029

expected p.v. of profit : 1688.90 **[2]**

[16 Marks]

****