### barrie+hibbert

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## Why is equity diversification absent during equity market stress events?

Understanding & modelling equity tail dependence

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## Agenda

- + Overview
- + The 2008 failure of diversification
- + Correlation and dependence
- + An insight from textbook portfolio theory
- + A better way for the future?



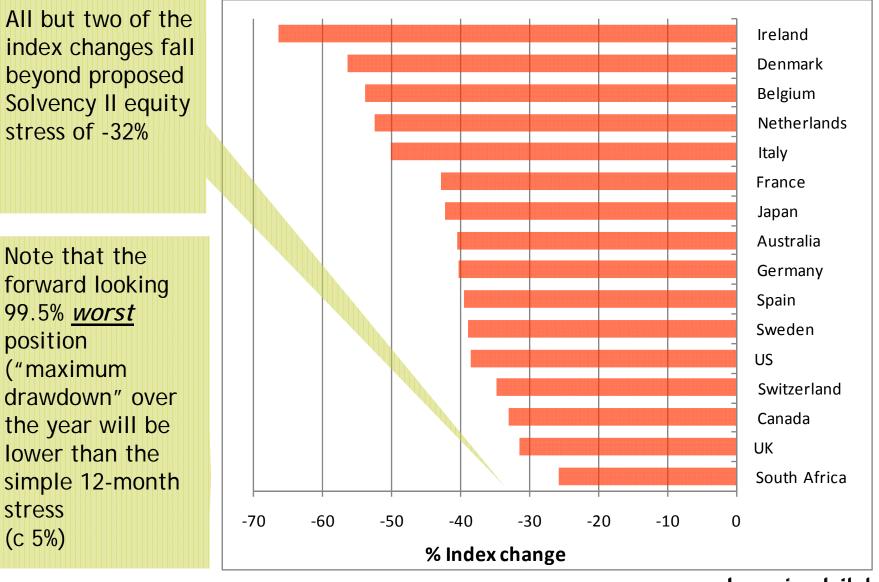
## Overview

- + Modern capital regimes require firms to set aside capital to survive extreme events.
- + The assessment of these extreme possibilities is an inherently difficult task because they are only rarely seen.
  - Some of the possibilities that <u>should</u> be analyzed will never have been observed in the past.
- + The year 2008 has been a stern test of firms and their financial models which a number of major firms have failed to survive.
- + Important questions have been raised about the value of models and their calibration (parameter choice).
- + Our focus is on the severity of equity market declines and the <u>extraordinary failure of diversification</u>.
- + We will argue that this strong dependence was foreseeable and can be incorporated relatively easily into models.

## The 2008 failure of diversification

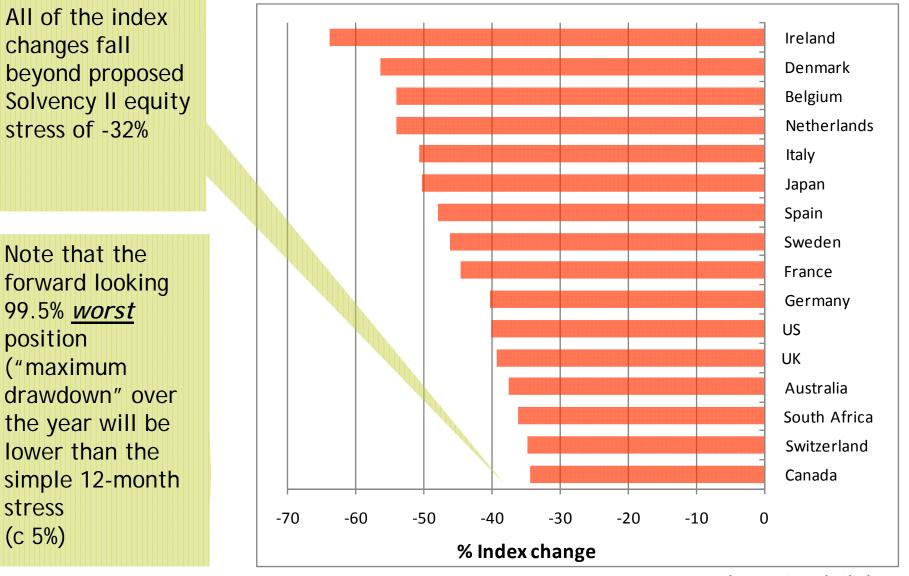
## Year-to-date equity index changes

#### (to 31 Dec 2008)



## Year-to-date equity index changes

### (to 28 Oct 2008)



## Analysis of historic equity tails

Pooled DMS tail excess returns to end-2007

#### 2002 Swi 2001 Ita 1992 Jap 1990 Swi 2002 Net 1990 Fra 2002 Ire 1990 Aus In 1931, 10 markets of 2002 Fra 1987 Swi the 16 delivered excess 1992 Den 1984 Den 1990 Spa 1982 Aus returns of -25% or 1990 Bel 1981 Can worse. 1987 Ita 1974 Ita 1987 Fra 1974 Fra 1977 Ita 1974 Can In 1974, 9 markets .... 1976 Spa 1973 Jap 1974 US 1973 Aus 1974 Bel 1970 Ger In 2002, 6 markets .... 1974 Aus 1970 Aus 1966 Bel 1973 UK 1948 Spa 1964 Ita How do the 2008 price 1947 Bel 1952 Ire 1937 US 1952 Aus changes compare? 1932 Spa 1947 Ita 2002 Swe 1932 Net 1941 Net 1990 Swe 1931 Swi 1931 UK 1990 Ire 1931 Ger 1931 Net 1987 Ger 1931 Fra 1931 Ita 1977 Spa 1931 Can 1930 Aus 1974 Swi 1930 US 1921 Swe 1946 Jap 1930 Can 1920 Swi 1931 Swe 1920 Sou 1920 Net 1974 Ire 2002 Ger 1945 Ita 1990 Jap 1931 Bel 1914 Swe 1919 Swi 1907 US 1974 UK 1920 Jap 1931 US 1926 Ita 1908 Jap -75% or -70% to -65% to -60% to -55% to -50% to -45% to -40% to -35% to -30% to -25% -65% -60% -55% -50% -45% -40% -35% -30% worse

## Analysis of historic equity tails

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## Analysis of historic equity tails

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worse	-65%	-60%	-55%	-50%	-45%	-40%	-35%	-30%	-25% <b>k</b>	parrie+hibbert

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## Correlation & dependence

# Average 10-day correlation across US, UK, Japan, Germany & France

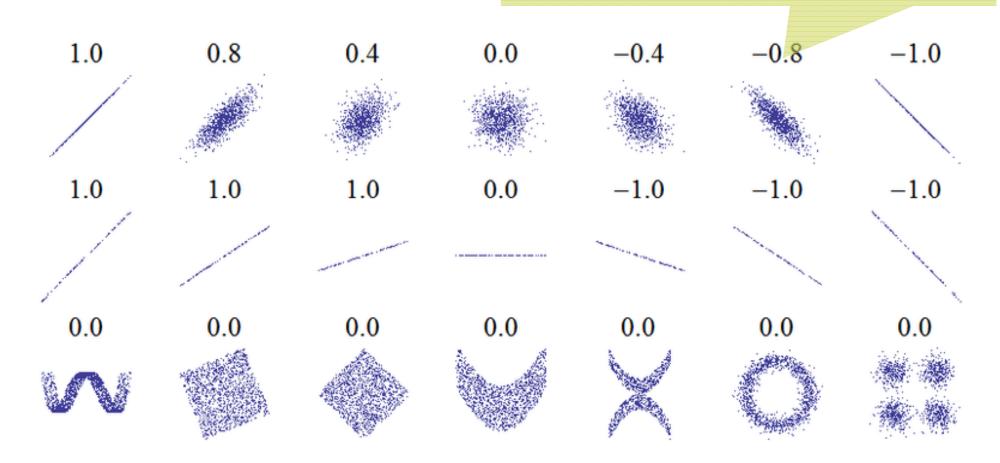
- + The correlation coefficient is the statistician's standard tool for describing dependence.
- + Average 10-day correlation across US, UK, Japan, Germany & France was 87% over 2008 (to end October 2008).
- + Average 'unconditional' equity correlation remains c0.50

	Japan	France	UK	Germany
Japan				
France	85%			
UK	83%	95%		
Germany	86%	95%	91%	
US	80%	84%	85%	84%
<b>A</b>	07.00/			

Average = 87.0%

### How much does the correlation coefficient tell us? Correlation does not unique dependency structure - di

Correlation does not uniquely specify the dependency structure - different joint distributions can have the same correlation.

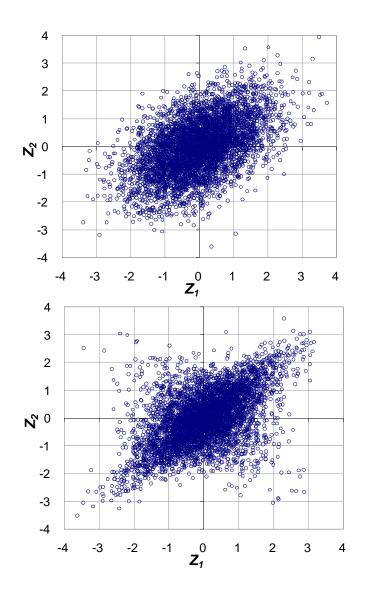


Source: <u>http://en.wikipedia.org/wiki/File:Correlation\_examples.png</u>, Released into the public domain (by the author)

## Copulas

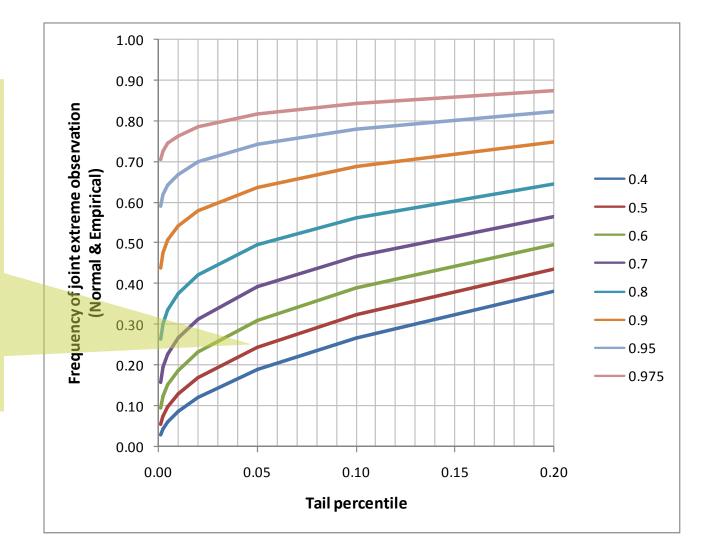
+ Copula methods offer the prospect of:

- capturing the entire dependency structure
- an ability to separate the description of dependency from the '*marginal*' distributions.
- + Compare *5000* sample random variates drawn from the same marginal distribution (Gaussian) and sharing the same linear correlation (*0.50*) but which exhibit different dependency in the tails:
  - *"Gaussian Copula"* = Dependency structure of the multivariate normal distribution (top chart)
  - *"t-Copula"* = Dependency structure of the multivariate t distribution (bottom chart).



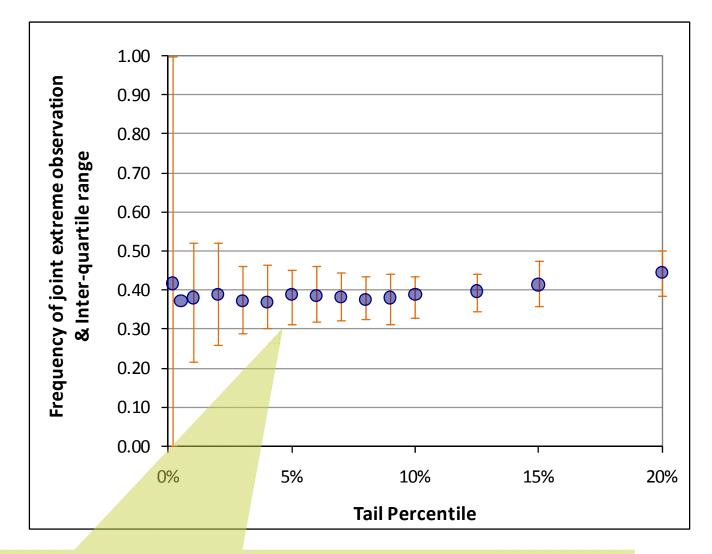
### A Measure of Tail Dependency Probability (Z1 & Z2 < j<sup>th</sup> percentile | Z1 < j<sup>th</sup> percentile)

If returns are correlated 0.50, under the Normal model the probability of observing a joint event beyond the 95<sup>th</sup> %tile (given one observation) is 25%



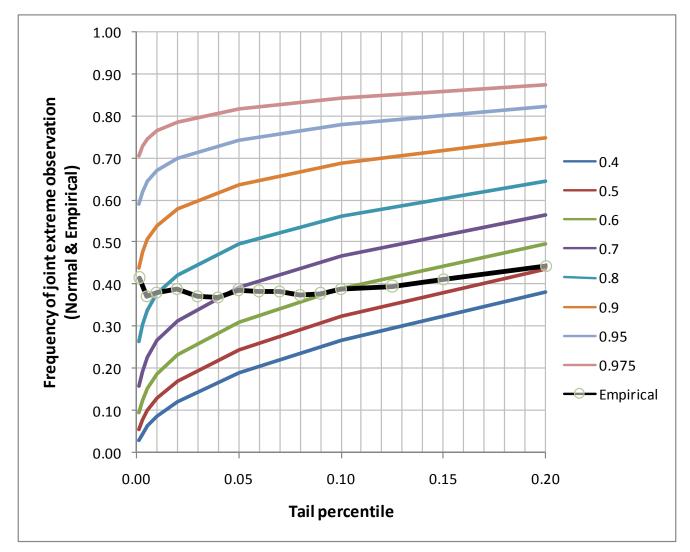
## **Empirical tail dependency**

Monthly price changes for large equity markets, 1958 to 2006



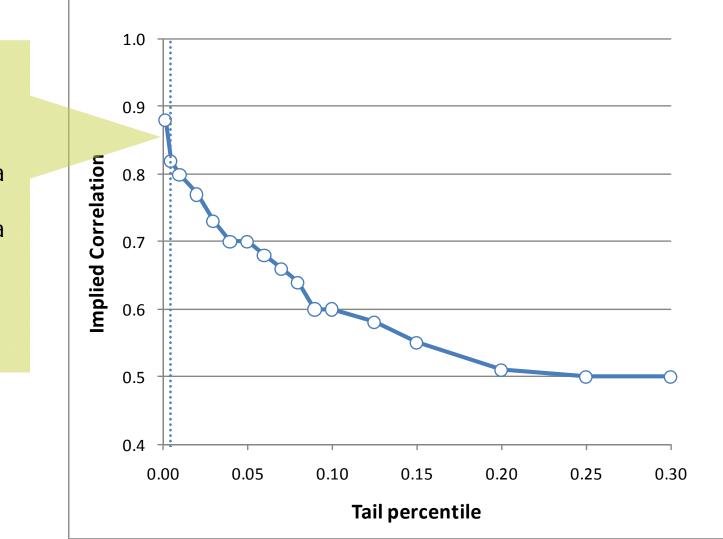
Tail dependence of 0.40 appears to be a reasonable estimate across bottom quartile

# A comparison of empirical tail dependency with the Normal copula



# What is the correlation 'implied' for the tail positions?

So the dependency observed at the 99.5 %tile is consistent with a *Normal* model calibrated with a correlation of c0.85



## An insight from textbook portfolio theory

## Single-index models - a reminder

The return on an individual asset *i* can be expressed as:

 $R_i = \alpha_i + \beta_i R_m$ 

where :

- a = the component of asset i's return which is independent of the common (index) return.
- $\beta_i$  = the sensitivity of asset i to the common "market" factor.
- $R_m$  = the return on the market (common factor) index.

The variance of the return on each asset is expressed in terms of the variance of the market, the asset's sensitivity to the market and each asset's residual (i.e. non-market) variance as follows:

$$\sigma_{i tot}^{2} = \beta_{i}^{2} \sigma_{M}^{2} + \sigma_{i spec}^{2}$$

where :

 $\beta_i$ 

 $\sigma^2_M$ 

 $\sigma_{i spec}^{2}$ 

- $\sigma_{i tot}^{2}$  = the total variance of returns on asset *i*.
  - = the sensitivity of asset *i* to the common "market" factor.
    - = the variance of the common "market" factor.
    - = the individual, specific variance of asset *i*.

## Single-index models - a reminder

Since the covariance between any pair of assets  $(\sigma_{ij})$  only arises as a result of their exposure to the shared common factor, we can express covariance as follows:

$$\sigma_{ij} = \beta_i \beta_j \sigma^2_M$$

The correlation between any pair of assets  $(r_{ij})$  can be expressed in terms of covariance and total asset variance as follows:

$$r_{ij} = \sigma_{ij} / \sigma_{i \ tot} \sigma_{j \ tot} = \beta_i \beta_j \sigma_M^2 / \sigma_{i \ tot} \sigma_{j \ tot}$$

### Single-index models - a reminder

Assume that the exposure to the common factor is the same for all assets so that  $\beta_i = 1$  for all *i*. Further suppose that all assets have identical total variance,  $\sigma_{tot}^2$  and residual variance,  $s_{spec}^2$ . In which case we can write for every asset:

$$\sigma_{i tot}^{2} = \sigma_{M}^{2} + \sigma_{i spec}^{2}$$

the correlation between every pair of assets, *r* is given by:

$$r = \beta_{i} \beta_{j} \sigma_{M}^{2} / \sigma_{i \ tot} \sigma_{j \ tot}$$
  
=  $\sigma_{M}^{2} / \sigma_{tot}^{2}$   
=  $1 - \sigma_{spec}^{2} / \sigma_{tot}^{2}$   
So if  $\sigma_{i \ tot}^{2} = 0.20^{2}$  and  $r = 0.50$ , then  
 $\sigma_{i \ spec}^{2} = \sigma_{tot}^{2} (1 - r) = 0.20^{2} (1 - 0.50) = 0.141^{2}$   
 $\sigma_{M}^{2} = \sigma_{i \ tot}^{2} - \sigma_{i \ spec}^{2} = 0.20^{2} - 0.141^{2} = 0.141^{2}$ 

## Single-index models

### Impact of elevation of common factor risk

Assume that all of the increase in variance has been caused by an increase in variance of the common factor return, and that the specific variance of equity markets is unchanged, so that:

$$\sigma_{M}^{2} = \sigma_{i \ tot}^{2} - \sigma_{i \ spec}^{2}$$

$$= 0.30^{2} - 0.141^{2}$$

$$= 0.265^{2}$$

$$r = \sigma_{M}^{2} / \sigma_{tot}^{2}$$

$$= 0.265^{2} / 0.30^{2}$$

$$= 0.78$$

## **Global factor risk & correlation**

Global Factor Risk	Specific Risk	Total Risk	Average Correlation
14	14	20	0.50
0	14	14	0.00
5	14	15	0.11
10	14	17	0.33
15	14	21	0.53
20	14	24	0.67
25	14	29	0.76
30	14	33	0.82
35	14	38	0.86
40	14	42	0.89
45	14	47	0.91
50	14	52	0.93

# Why have firms not adopted this approach as standard practice?

- + These ideas are hardly new and many practitioners would view them as intuitive.
- + Nevertheless, there are reasons why insurers might not have chosen to model equity market risk in this way:
  - Cost
  - Technical implementation challenges

## An example of the capital cost of stronger assumptions for tail dependence

Correlation	Portfolio 99.5% stress	Required Capital	Additional Capital vs 'base' 50% correlation
0.50	-0.357	0.554	
0.75	-0.397	0.658	18.7%
0.80	-0.404	0.679	22.4%
0.85	-0.412	0.699	26.1%
0.90	-0.418	0.720	29.8%
0.95	-0.425	0.740	33.4%

Note: Assumed portfolio assets have equal arithmetic expected return of 8% & volatility of 23% pa with weights of 50%-25%-12.5%-12.5%.

## Conclusions & way forward

- You need to model a global common factor with stochastic volatility in order to capture realistic behaviour of equity markets in (some) times of stress.
- + Remember that this dependence probably extends beyond equity markets to real estate and credit exposures.
- A further ingredient in equity modelling is the possibility of 'jumps' in prices.
- + This mix of uncertain (Stochastic) Volatility and Jump Diffusion (SVJD) is a powerful tool for risk modellers and can be calibrated both to:
  - Market option prices capturing the complex smiles, skews and term structures seen in these markets.
  - To produce plausible real-world scenarios for projection purposes.

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