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February 2009: Global Conference of Actuaries

Why is equity diversification absent during equity market stress events?

Understanding & modelling equity tail dependence

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Agenda

- Overview
- The 2008 failure of diversification
- Correlation and dependence
- An insight from textbook portfolio theory
- A better way for the future?

Overview

Overview

- Modern capital regimes require firms to set aside capital to survive extreme events.
- The assessment of these extreme possibilities is an inherently difficult task because they are only rarely seen.
  - Some of the possibilities that should be analyzed will never have been observed in the past.
- The year 2008 has been a stern test of firms and their financial models which a number of major firms have failed to survive.
- Important questions have been raised about the value of models and their calibration (parameter choice).
- Our focus is on the severity of equity market declines and the *extraordinary failure of diversification*.
- We will argue that this strong dependence was foreseeable and can be incorporated relatively easily into models.

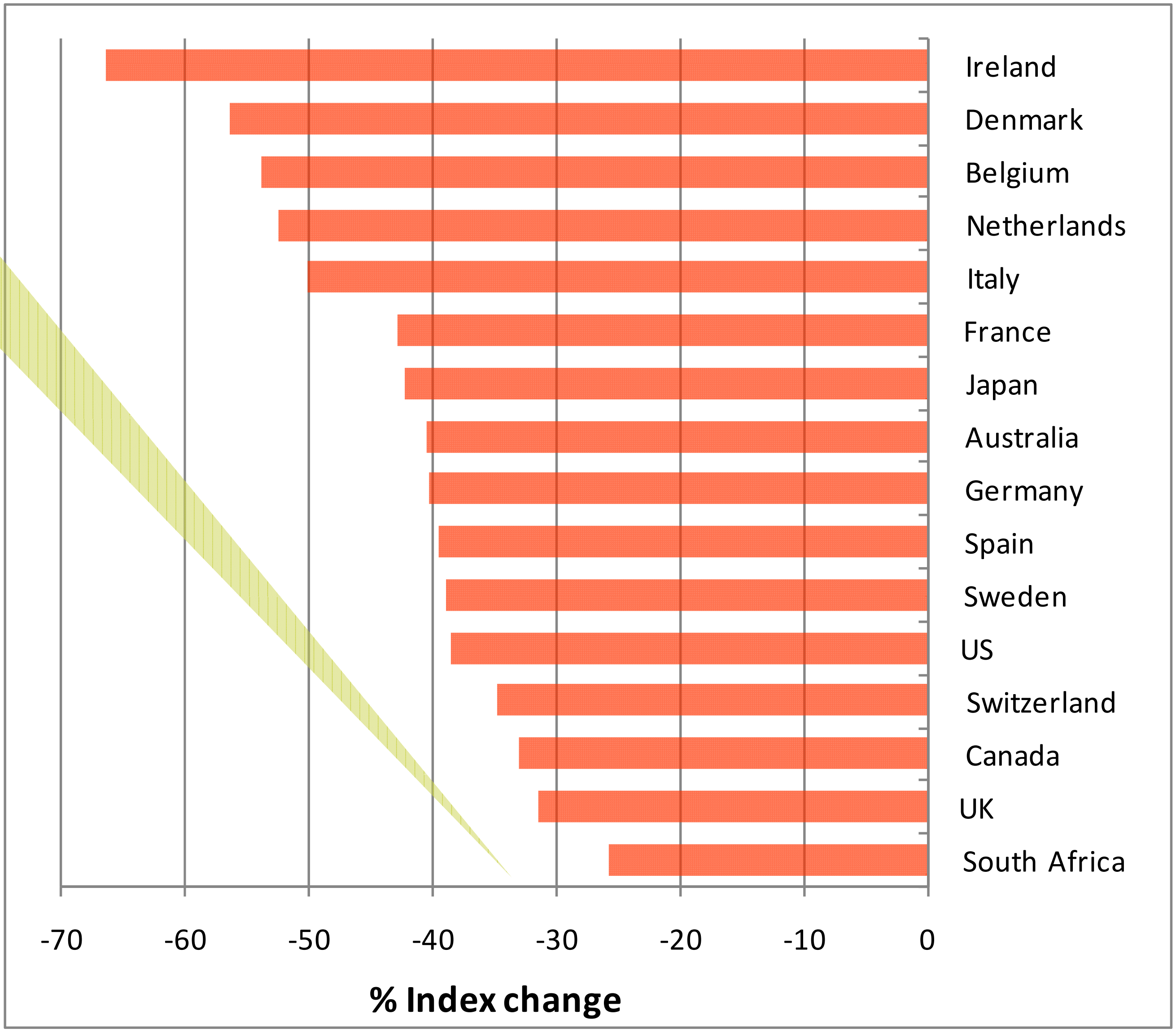
The 2008 failure of diversification

Year-to-date equity index changes

(to 31 Dec 2008)

All but two of the index changes fall beyond proposed Solvency II equity stress of -32%

Note that the forward looking 99.5% ***worst*** position (“maximum drawdown” over the year will be lower than the simple 12-month stress (c 5%)

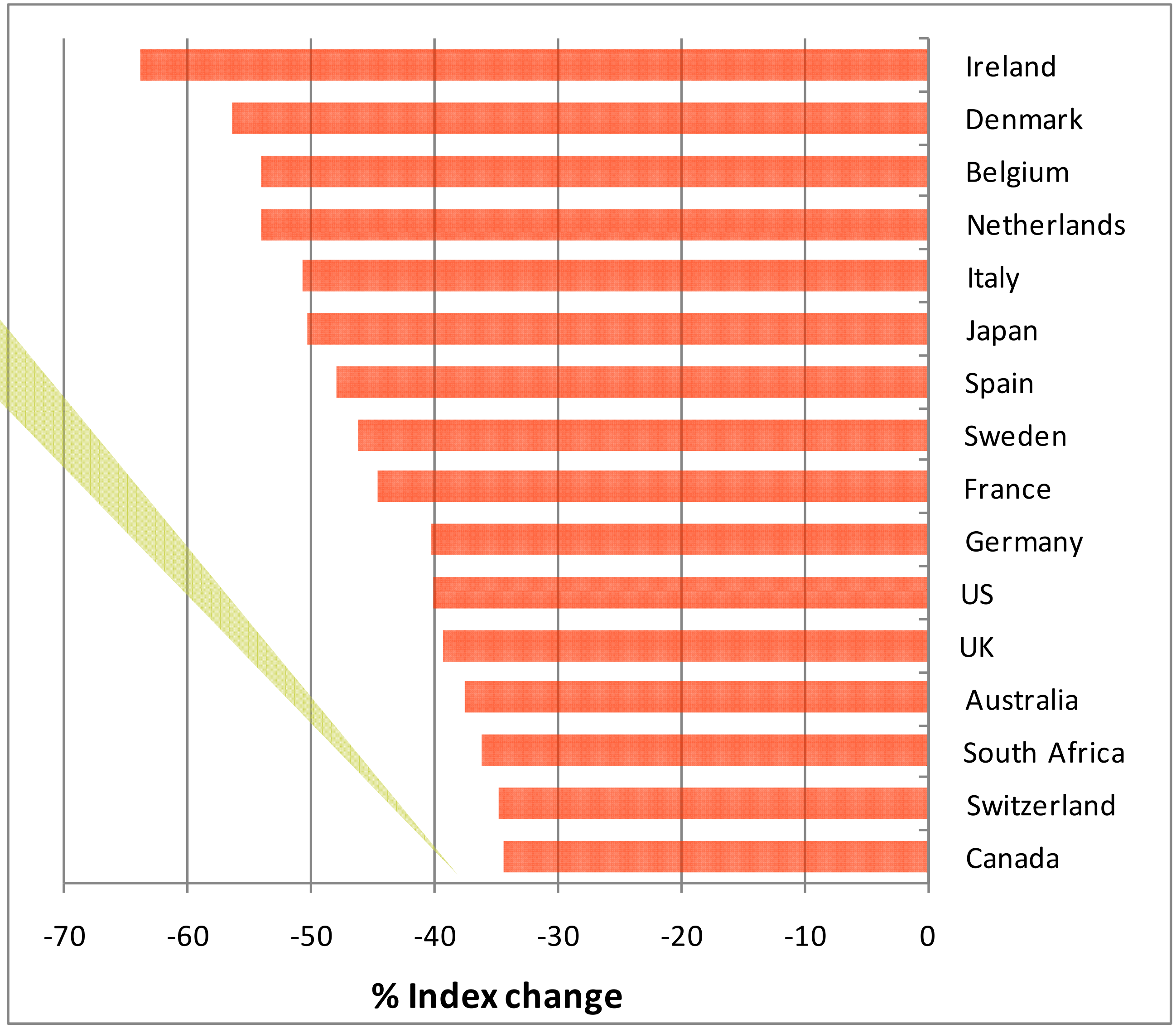


Year-to-date equity index changes

(to 28 Oct 2008)

All of the index changes fall beyond proposed Solvency II equity stress of -32%

Note that the forward looking 99.5% ***worst*** position ("maximum drawdown" over the year will be lower than the simple 12-month stress (c 5%)



Analysis of historic equity tails

Pooled DMS tail excess returns to end-2007

In 1931, 10 markets of the 16 delivered excess returns of -25% or worse.

In 1974, 9 markets ….

In 2002, 6 markets ….

How do the 2008 price changes compare?

-75% or worse	-70% to -65%	-65% to -60%	-60% to -55%	-55% to -50%	-50% to -45%	-45% to -40%	-40% to -35%	-35% to -30%	-30% to -25%
									2002 Swi
									2001 Ita
									1992 Jap
									1990 Swi
								2002 Net	1990 Fra
								2002 Ire	1990 Aus
								2002 Fra	1987 Swi
								1992 Den	1984 Den
								1990 Spa	1982 Aus
								1990 Bel	1981 Can
								1987 Ita	1974 Ita
								1987 Fra	1974 Fra
								1977 Ita	1974 Can
								1976 Spa	1973 Jap
								1974 US	1973 Aus
								1974 Bel	1970 Ger
								1974 Aus	1970 Aus
								1973 UK	1966 Bel
								1948 Spa	1964 Ita
								1947 Bel	1952 Ire
								1937 US	1952 Aus
								1932 Spa	1947 Ita
							2002 Swe	1932 Net	1941 Net
							1990 Swe	1931 Swi	1931 UK
							1990 Ire	1931 Ger	1931 Net
							1987 Ger	1931 Fra	1931 Ita
							1977 Spa	1931 Can	1930 Aus
							1974 Swi	1930 US	1921 Swe
							1946 Jap	1930 Can	1920 Swi
					1974 Ire	2002 Ger	1931 Swe	1920 Sou	1920 Net
					1945 Ita	1990 Jap	1931 Bel	1914 Swe	1919 Swi
			1974 UK		1920 Jap	1931 US	1926 Ita	1907 US	1908 Jap

Analysis of historic equity tails

Pooled excess returns & 2008 price changes

Index price changes for all 16 equity markets fall below -25%.

The year-to-date equity falls are truly exceptional.

-70% or worse	-70% to -65%	-65% to -60%	-60% to -55%	-55% to -50%	-50% to -45%	-45% to -40%	-40% to -35%	-35% to -30%	-30% to -25%
									2008 Sou
									2007 Ire
									2002 Swi
								2008 UK	2001 Ita
								2008 Swi	1992 Jap
								2008 Can	1990 Swi
								2002 Net	1990 Fra
								2002 Ire	1990 Aus
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								1992 Den	1984 Den
								1990 Spa	1982 Aus
								1990 Bel	1981 Can
								1987 Ita	1974 Ita
								1987 Fra	1974 Fra
								1977 Ita	1974 Can
								1976 Spa	1973 Jap
								1974 US	1973 Aus
								1974 Bel	1970 Ger
								1974 Aus	1970 Aus
								1973 UK	1966 Bel
								1948 Spa	1964 Ita
							2008 US	1947 Bel	1952 Ire
							2008 Spa	1937 US	1952 Aus
							2008 Swe	1932 Spa	1947 Ita
							2002 Swe	1932 Net	1941 Net
							1990 Swe	1931 Swi	1931 UK
							1990 Ire	1931 Ger	1931 Net
						2008 Jap	1987 Ger	1931 Fra	1931 Ita
						2008 Ger	1977 Spa	1931 Can	1930 Aus
						2008 Fra	1974 Swi	1930 US	1921 Swe
					2008 Ita	2008 Aus	1946 Jap	1930 Can	1920 Swi
					1974 Ire	2002 Ger	1931 Swe	1920 Sou	1920 Net
			2008 Den	2008 Net	1945 Ita	1990 Jap	1931 Bel	1914 Swe	1919 Swi
	2008 Ire		1974 UK	2008 Bel	1920 Jap	1931 US	1926 Ita	1907 US	1908 Jap

Analysis of historic equity tails

Pooled excess returns & 2008 price change to 28 Oct

To 28th October, index price changes for all 16 equity markets fall below -30%.

The year-to-date equity falls are truly exceptional, but the year is not finished yet.

-75% or worse	-70% to -65%	-65% to -60%	-60% to -55%	-55% to -50%	-50% to -45%	-45% to -40%	-40% to -35%	-35% to -30%	-30% to -25%
									2002 Swi
									2001 Ita
								2008 Swi	1992 Jap
								2008 Can	1990 Swi
								2002 Net	1990 Fra
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								1974 US	1973 Aus
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								1948 Spa	1964 Ita
							2008 UK	1947 Bel	1952 Ire
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							1990 Ire	1931 Ger	1931 Net
							1987 Ger	1931 Fra	1931 Ita
						2008 US	1977 Spa	1931 Can	1930 Aus
					2008 Swe	2008 Ger	1974 Swi	1930 US	1921 Swe
				2008 Net	2008 Spa	2008 Fra	1946 Jap	1930 Can	1920 Swi
				2008 Jap	1974 Ire	2002 Ger	1931 Swe	1920 Sou	1920 Net
	2008 Rus		2008 Den	2008 Ita	1945 Ita	1990 Jap	1931 Bel	1914 Swe	1919 Swi
	2008 Chi	2008 Ire	1974 UK	2008 Bel	1920 Jap	1931 US	1926 Ita	1907 US	1908 Jap

Correlation & dependence

Average 10-day correlation across US, UK, Japan, Germany & France

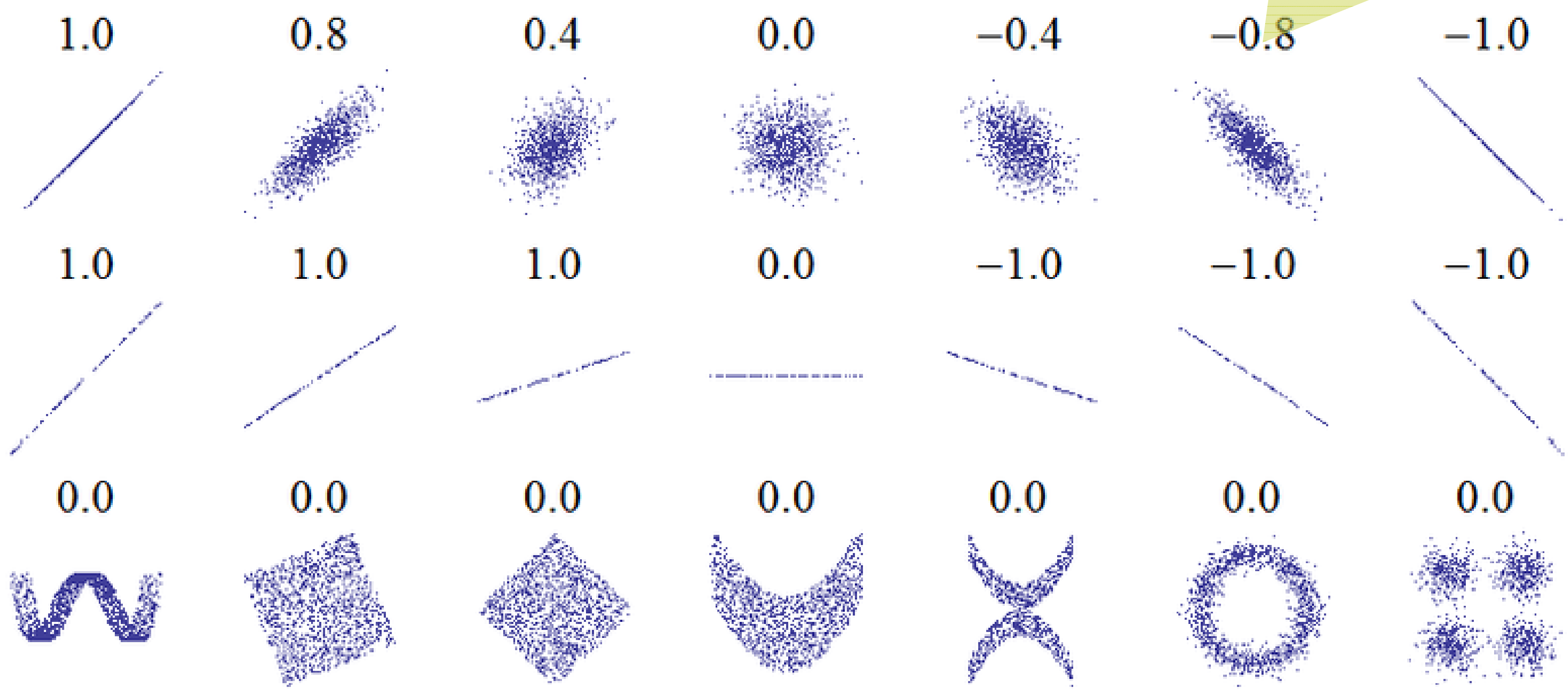
+ The correlation coefficient is the statistician's standard tool for describing dependence.
+ Average 10-day correlation across US, UK, Japan, Germany & France was 87% over 2008 (to end October 2008).
+ Average 'unconditional' equity correlation remains c*0.50*

	Japan	France	UK	Germany
Japan				
France	85%			
UK	83%	95%		
Germany	86%	95%	91%	
US	80%	84%	85%	84%

Average = 87.0%

How much does the correlation coefficient tell us?

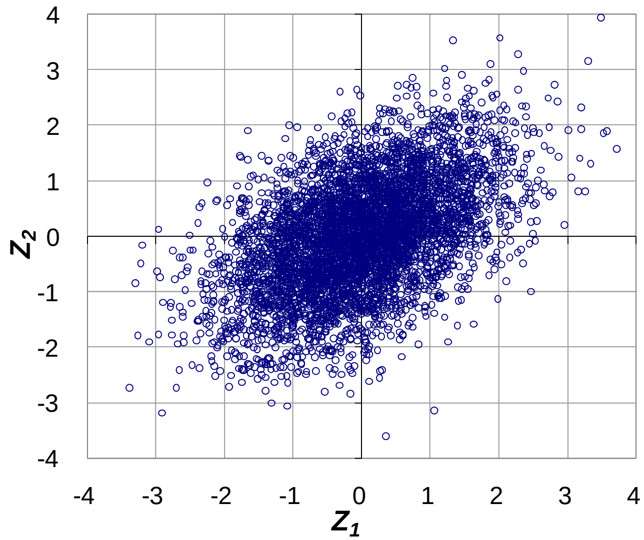
Correlation does not uniquely specify the dependency structure - different joint distributions can have the same correlation.

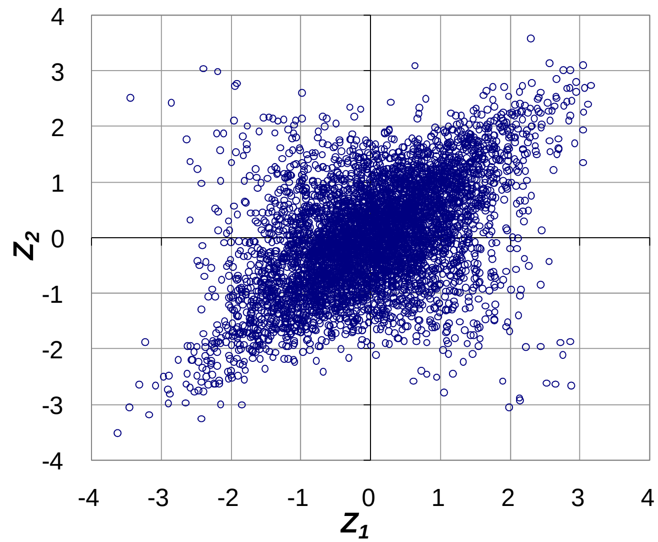


Source: http://en.wikipedia.org/wiki/File:Correlation_examples.png, Released into the public domain (by the author)

Copulas

- Copula methods offer the prospect of:
  - capturing the entire dependency structure
  - an ability to separate the description of dependency from the '*marginal*' distributions.
- Compare *5000* sample random variates drawn from the same marginal distribution (Gaussian) and sharing the same linear correlation (*0.50*) but which exhibit different dependency in the tails:
  - "*Gaussian Copula*" = Dependency structure of the multivariate normal distribution (top chart)
  - "*t-Copula*" = Dependency structure of the multivariate t distribution (bottom chart).

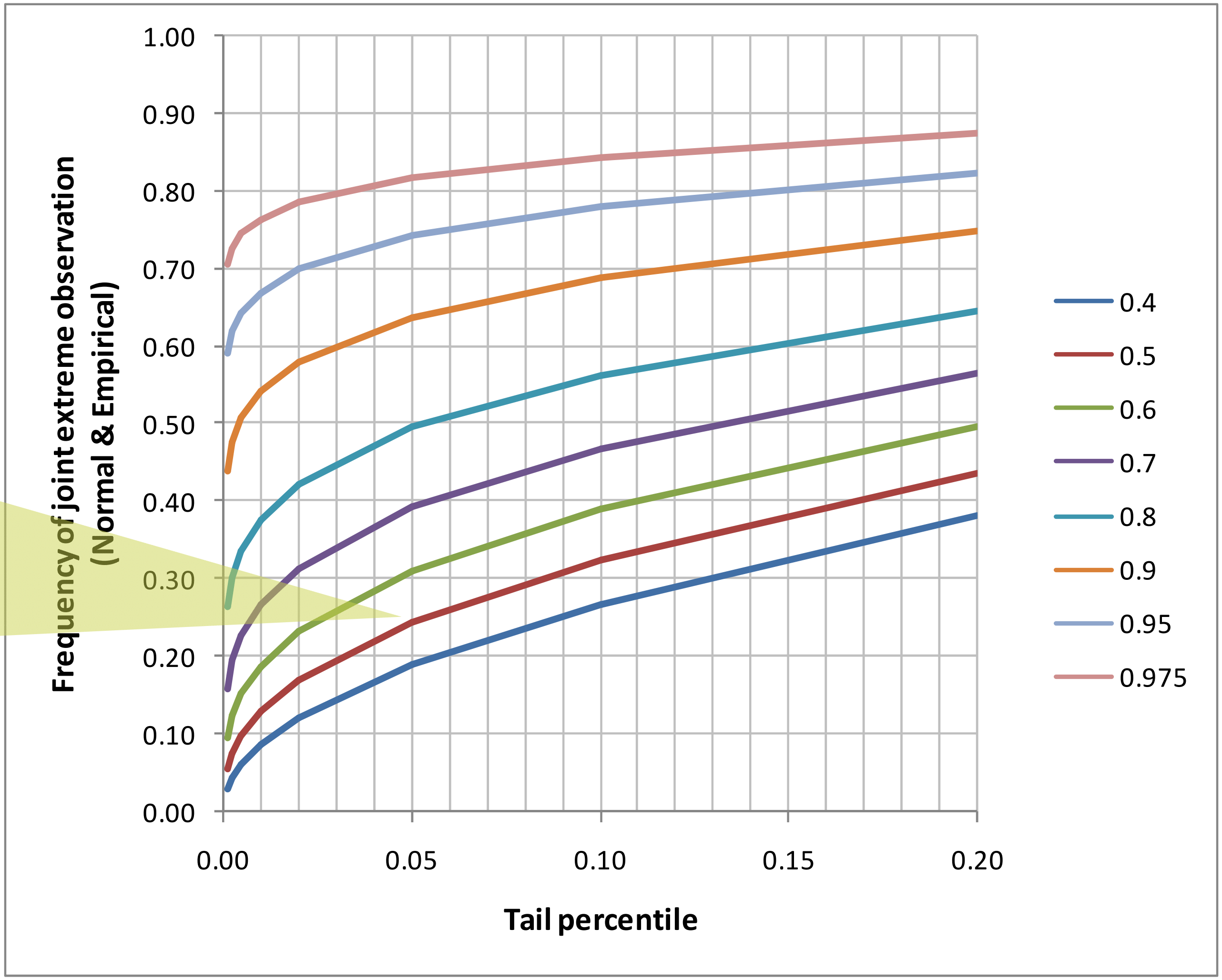




A Measure of Tail Dependency

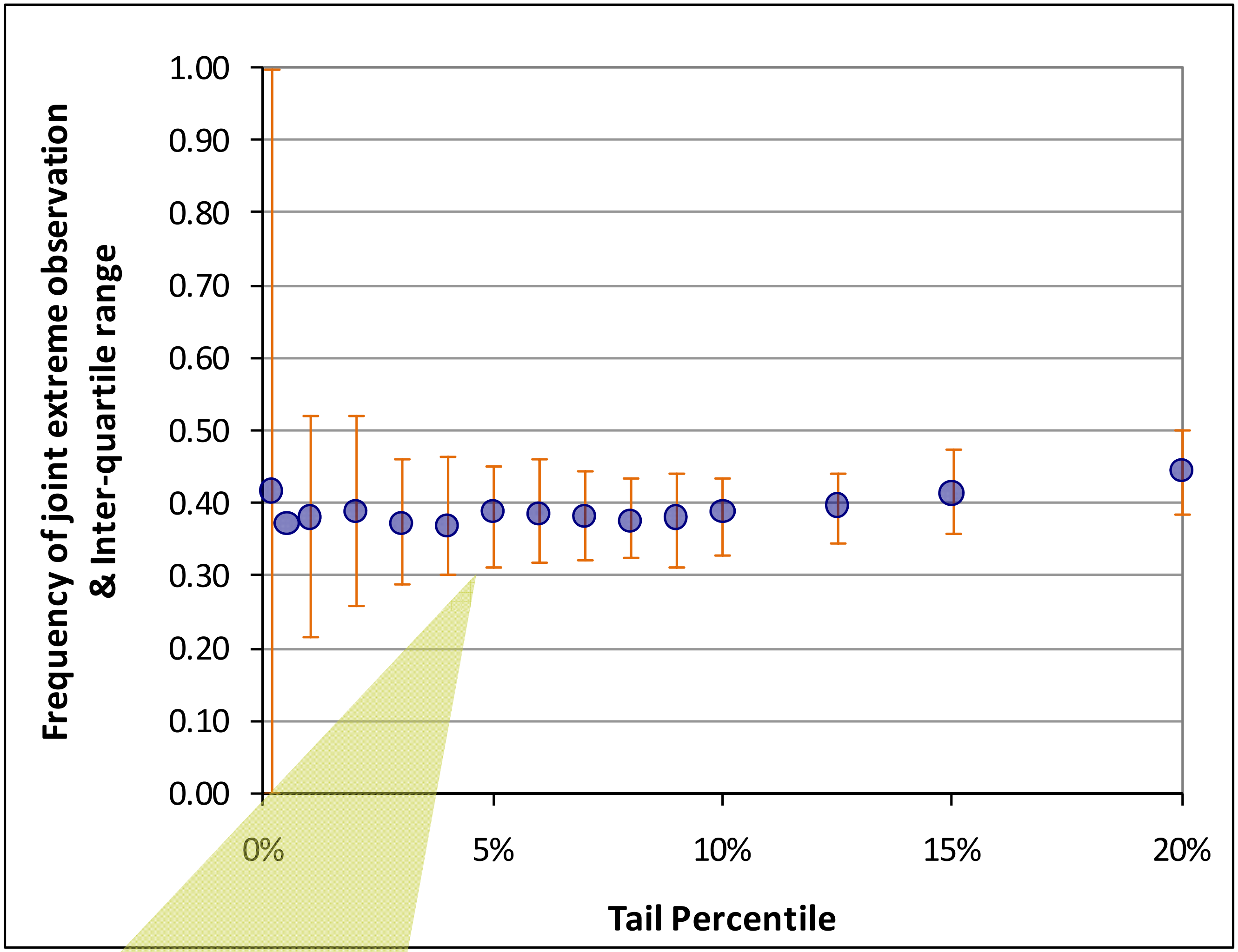
Probability (Z1 & Z2 < j^{th} percentile | Z1 < j^{th} percentile)

If returns are correlated 0.50, under the Normal model the probability of observing a joint event beyond the 95th %tile (given one observation) is 25%



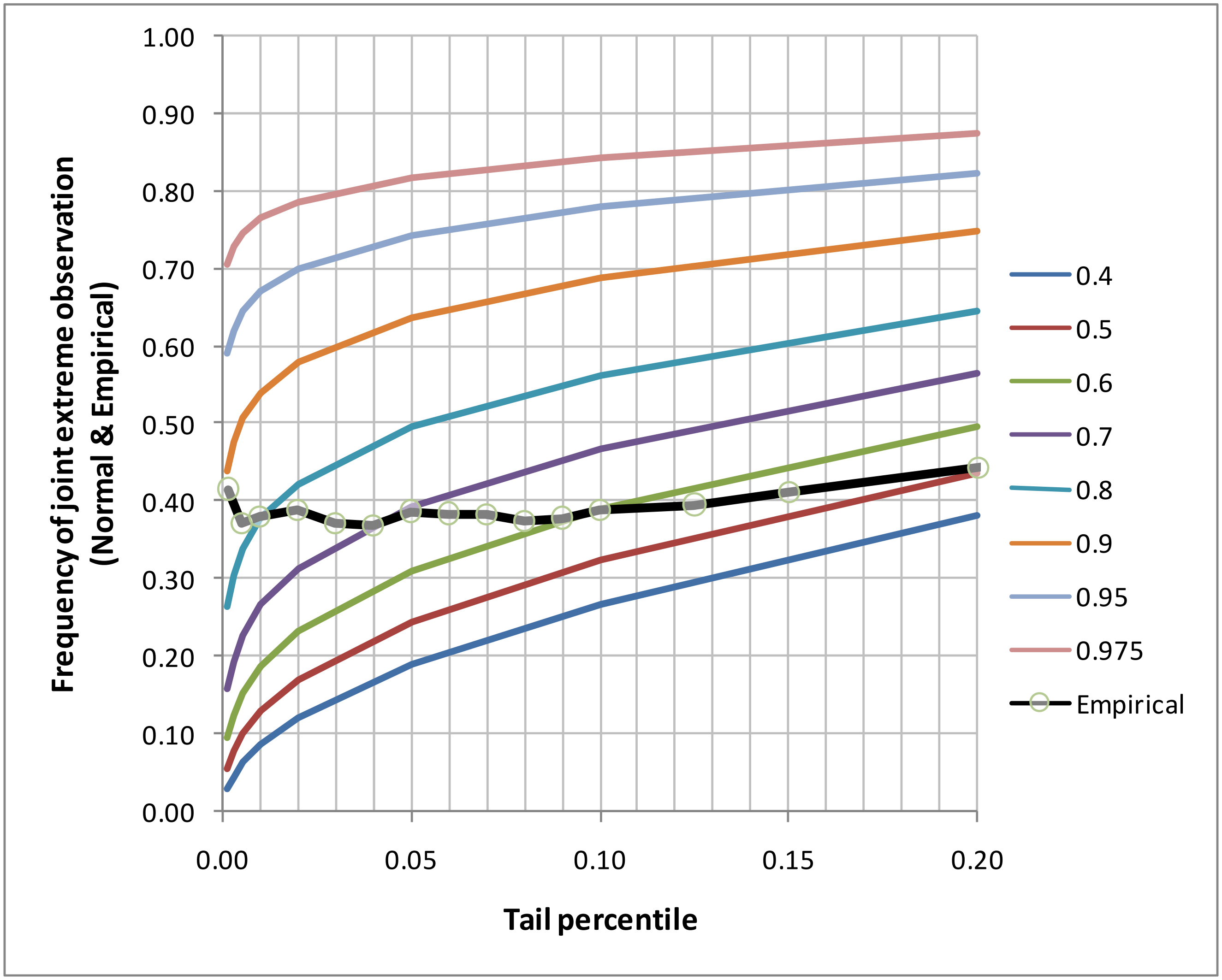
Empirical tail dependency

Monthly price changes for large equity markets, 1958 to 2006



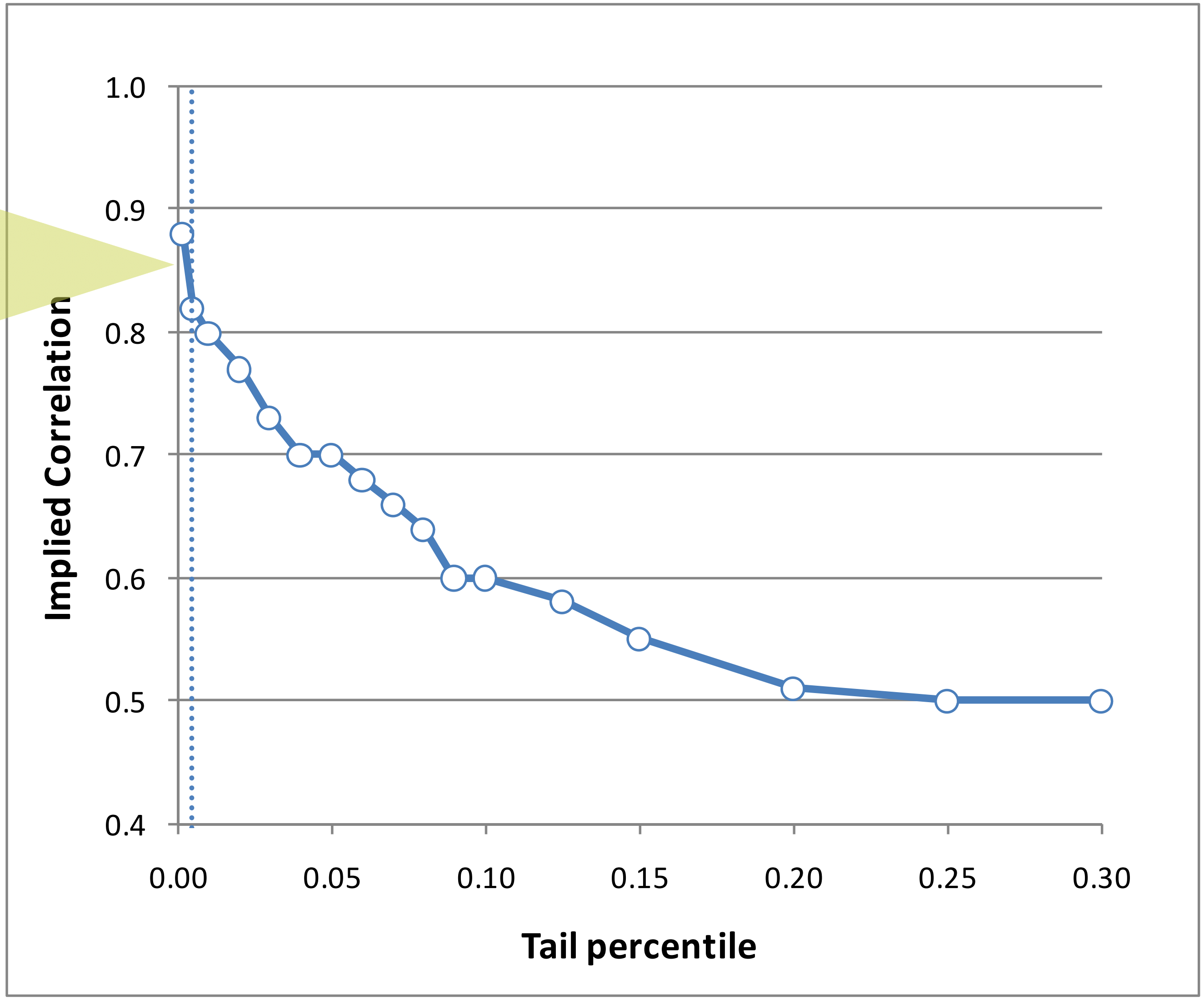
Tail dependence of 0.40 appears to be a reasonable estimate across bottom quartile

A comparison of empirical tail dependency with the Normal copula



What is the correlation ‘implied’ for the tail positions?

So the dependency observed at the 99.5 %tile is consistent with a *Normal* model calibrated with a correlation of c0.85



An insight from textbook portfolio theory

Single-index models - a reminder

The return on an individual asset *i* can be expressed as:

$R_i = \alpha_i + \beta_i R_m$

where :

α_i = the component of asset i's return which is independent of the common (index) return.

β_i = the sensitivity of asset i to the common "market" factor.

R_m = the return on the market (common factor) index.

The variance of the return on each asset is expressed in terms of the variance of the market, the asset's sensitivity to the market and each asset's residual (i.e. non-market) variance as follows:

$\sigma_i^2{}_{tot} = \beta^2{}_i \sigma^2{}_M + \sigma_i^2{}_{spec}$

where :

$\sigma_i^2{}_{tot}$ = the total variance of returns on asset *i*.

β_i = the sensitivity of asset *i* to the common "market" factor.

$\sigma^2{}_M$ = the variance of the common "market" factor.

$\sigma_i^2{}_{spec}$ = the individual, specific variance of asset *i*.

Single-index models - a reminder

Since the covariance between any pair of assets (σ_{ij}) only arises as a result of their exposure to the shared common factor, we can express covariance as follows:

$$\sigma_{ij} \quad = \beta_i \beta_j \sigma^2_M$$

The correlation between any pair of assets (r_{ij}) can be expressed in terms of covariance and total asset variance as follows:

$$r_{ij} \quad = \sigma_{ij} \; / \; \sigma_{i\ tot}\, \sigma_{j\ tot}$$

$$= \beta_i \beta_j \sigma^2_M \; / \; \sigma_{i\ tot}\, \sigma_{j\ tot}$$

Single-index models - a reminder

Assume that the exposure to the common factor is the same for all assets so that $\beta_i = 1$ for all i. Further suppose that all assets have identical total variance, σ^2_{tot} and residual variance, s^2_{spec}. In which case we can write for every asset:

$$\sigma_i^2{}_{tot} = \sigma^2_M + \sigma_i^2{}_{spec}$$

the correlation between every pair of assets, r is given by:

$$r = \beta_i \beta_j \sigma^2_M / \sigma_{i\ tot}\, \sigma_{j\ tot}$$

$$= \sigma^2_M / \sigma^2_{tot}$$

$$= 1 - \sigma^2_{spec} / \sigma^2_{tot}$$

So if $\sigma_i^2{}_{tot} = 0.20^2$ and $r = 0.50,$ then

$$\sigma_i^2{}_{spec} = \sigma^2_{tot}\,(1 - r) = 0.20^2\,(1 - 0.50) = 0.141^2$$

$$\sigma^2_M = \sigma_i^2{}_{tot} - \sigma_i^2{}_{spec} = 0.20^2 - 0.141^2 = 0.141^2$$

Single-index models

Impact of elevation of common factor risk

Assume that all of the increase in variance has been caused by an increase in variance of the common factor return, and that the specific variance of equity markets is unchanged, so that:

$$\sigma^2_M = \sigma_i^2{}_{tot} - \sigma_i^2{}_{spec}$$

$$= 0.30^2 - 0.141^2$$

$$= 0.265^2$$

$$r = \sigma^2_M / \sigma^2_{tot}$$

$$= 0.265^2 / 0.30^2$$

$$= 0.78$$

Global factor risk & correlation

Global Factor Risk	Specific Risk	Total Risk	Average Correlation
14	14	20	0.50
0	14	14	0.00
5	14	15	0.11
10	14	17	0.33
15	14	21	0.53
20	14	24	0.67
25	14	29	0.76
30	14	33	0.82
35	14	38	0.86
40	14	42	0.89
45	14	47	0.91
50	14	52	0.93

Why have firms not adopted this approach as standard practice?

+ These ideas are hardly new and many practitioners would view them as intuitive.
+ Nevertheless, there are reasons why insurers might not have chosen to model equity market risk in this way:
  - Cost
  - Technical implementation challenges

An example of the capital cost of stronger assumptions for tail dependence

Correlation	Portfolio 99.5% stress	Required Capital	Additional Capital vs 'base' 50% correlation
0.50	-0.357	0.554	
0.75	-0.397	0.658	18.7%
0.80	-0.404	0.679	22.4%
0.85	-0.412	0.699	26.1%
0.90	-0.418	0.720	29.8%
0.95	-0.425	0.740	33.4%

Note: Assumed portfolio assets have equal arithmetic expected return of 8% & volatility of 23% pa with weights of 50%-25%-12.5%-12.5%.

Conclusions & way forward

+ You need to model a global common factor with stochastic volatility in order to capture realistic behaviour of equity markets in (some) times of stress.
+ Remember that this dependence probably extends beyond equity markets to real estate and credit exposures.
+ A further ingredient in equity modelling is the possibility of 'jumps' in prices.
+ This mix of uncertain (Stochastic) Volatility and Jump Diffusion (SVJD) is a powerful tool for risk modellers and can be calibrated both to:
  - Market option prices – capturing the complex smiles, skews and term structures seen in these markets.
  - To produce plausible real-world scenarios for projection purposes.