

Empirical Study of effect of using Weibull
distribution in Black-Scholes Formula on
NIFTY index options

Harsh Narang
Vatsala Deora

11th Global Conference of Actuaries

Overview

- ❑ NIFTY NSE index options
- ❑ Black Scholes model (lognormal distribution)
- ❑ Replacing the lognormal distribution with Weibull distribution
- ❑ Comparison on the basis of difference in prices
- ❑ Comparison on the basis of ease of use and implementation

NSE Nifty options

- ❑ S&P CNX Nifty (Nifty) is a 50-stock index comprising the largest and the most liquid companies in India
- ❑ Option trading began in June 2001
- ❑ Uses Black Scholes model (lognormal)
- ❑ A call option gives its holder the right to buy whereas put option gives its holder the right to sell
- ❑ Analysis based on call prices of Nifty Options (European type)

Black-Scholes model

- ❑ Cornerstone of option pricing since its inception
- ❑ Used in both financial literature as well as practice
- ❑ Arbitrary assumption of lognormal distribution
- ❑ Deficient in pricing deep in the money and deep out of the money options using statistical estimates of volatility.
- ❑ Still widely prevalent due to simplicity of the model and ease of use

Option pricing using lognormal distribution

- Under general asset pricing theory with the no-arbitrage condition, the current price of an asset is equal to the present value of its expected payoffs discounted at an appropriate rate.
- We assume that dividends are paid continuously, and that the dividend amount is proportional to the level of the index.

Black-Scholes Formula

$$C(S_0, T) = e^{-rt} (F\Phi(d_1) - K\Phi(d_2))$$

where

$$F = S_0 e^{(r-q)T}$$

is the modified forward price that occurs in the terms d_1 and d_2 :

$$d_1 = \frac{\ln\left(\frac{F}{K}\right) + \left(\frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \quad d_2 = d_1 - \sigma\sqrt{T}$$

S_0 : the price of the underlying security at time 0.

X : exercise price

r : the risk-free rate per annum with continuous compounding

σ^2 : variance rate of return for the underlying security

T : time to expiration

c : market value of call option at time 0

p : market value of put option at time 0

q : the dividend yield

$N(d)$: cumulative probability distribution function for a standardized normal distribution at d .

Limitations & Modifications

- ❑ Concept of implied volatility
- ❑ Value of implied volatility for different strike prices should theoretically be identical, but is usually seen to vary.
- ❑ The third central moment of the price distribution is significantly negative.
- ❑ Two solutions are possible:
 - Adjusting the lognormal distribution by a skewness–correction term
 - Using a distribution with natural negative skewness

Weibull Distribution

$$f(x) = \alpha\beta x^{b-1} e^{-\alpha x^\beta}, x \in (0, \infty) \alpha > 0, \beta > 0$$

- It is widely used in survival analysis & in extreme value theory
- Its skewness is negative for most parameter values which makes it more appropriate for valuing Nifty options

Option pricing using weibull distribution

- Derived using the method of equivalent martingale measures
- If it is assumed that the stock price at expiration, S_T , is distributed Weibull:

$$C = S_0 e^{-q\tau} \cdot \left[1 - F_{2(1+1/\beta)}(2\omega) \right] - K e^{-r\tau} \cdot e^{-\omega}$$

$F_d(a)$ denotes the cumulative distribution function at point a of χ_2 distributed random variable with degrees of freedom d and where

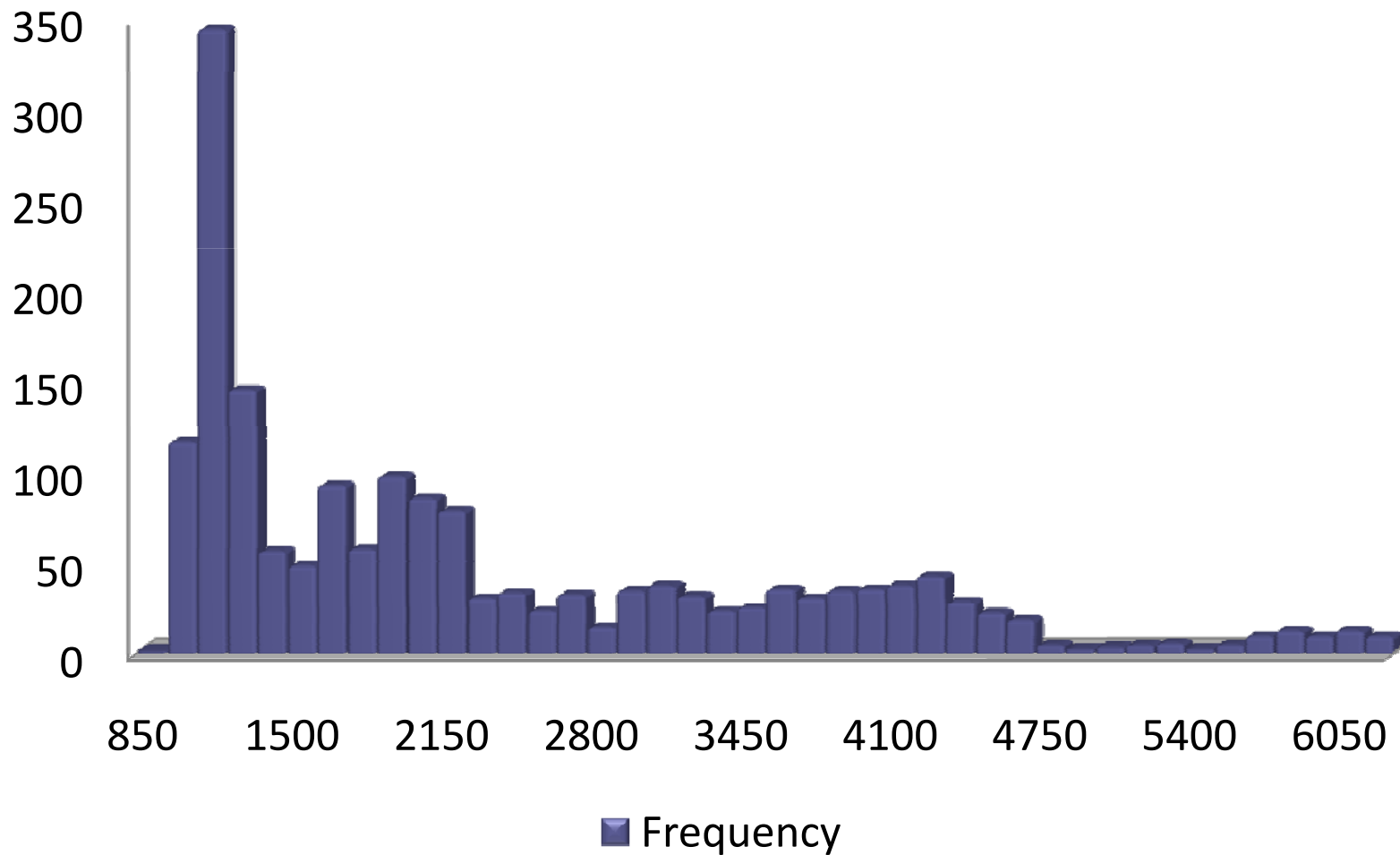
$$\omega = \alpha \cdot K^\beta e^{-\beta \cdot (r-q) \cdot \tau}$$

$$\alpha = \left[\frac{\Gamma(1+1/\beta)}{S_0} \right]^\beta \quad \beta = 1/p - 1$$

Data used for analysis

- ❑ The NSE nifty daily returns from 1st January 2001 to 1st January 2008 have been used.
- ❑ Then comparison between the two distributions was based on call prices of Nifty Options (European type) for the period of about three months from 3rd October 2007 to 27th December 2007.
- ❑ The daily MIBOR (Mumbai Inter Bank Offer Rate) rate has been taken as the risk free interest rate.
- ❑ Data for thinly traded options (less than 100 contracts on a given day) was excluded from the study.

NSE nifty Frequency Plot of Closing Values 01/01/2001 -01/01/2008

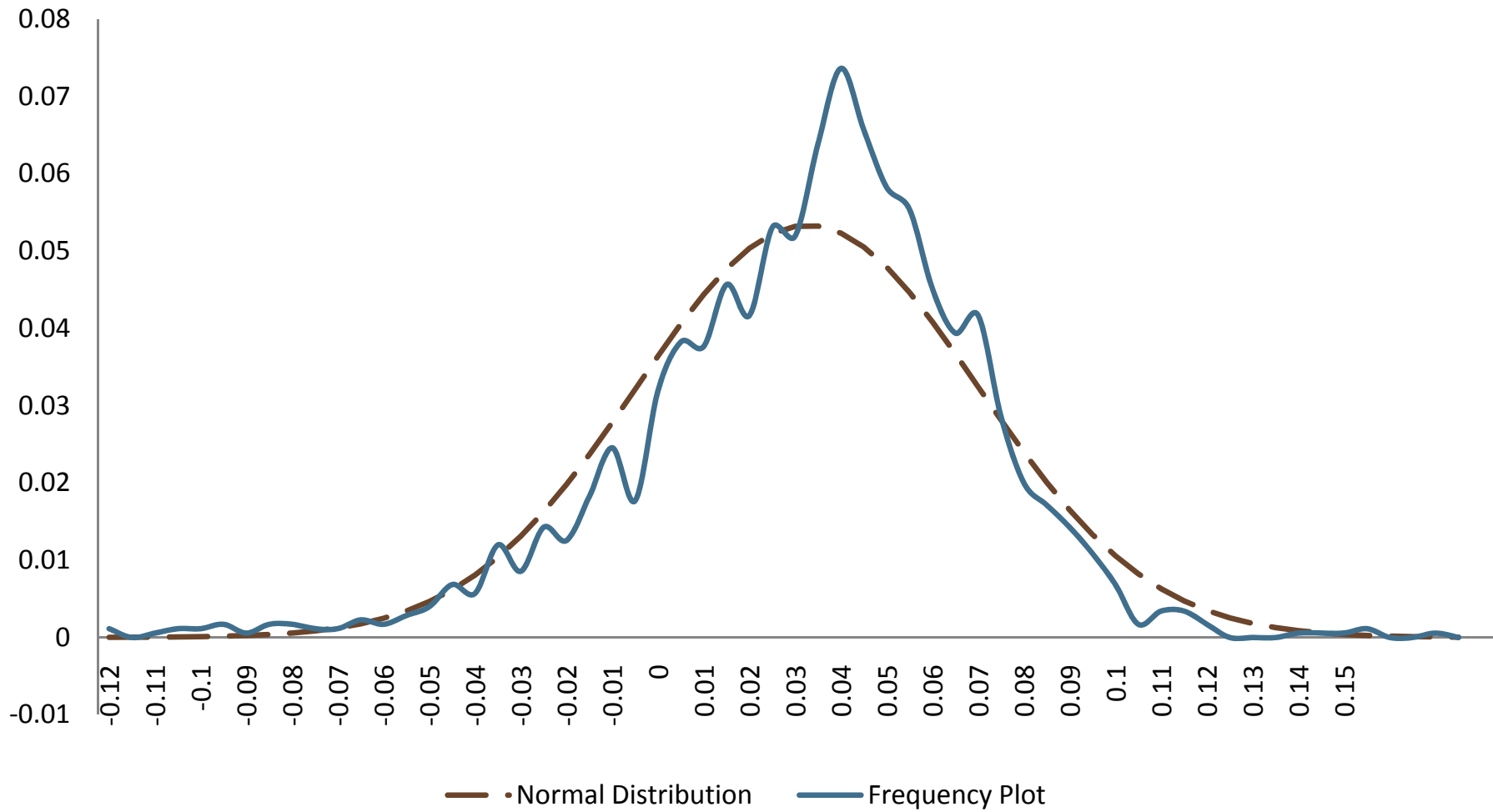


NSE nifty returns data

	Daily Returns	Weekly Returns
Mean	2.435315	16.09323
Variance	1542.746	10041.630
Std Error	39.27780	100.20793
Skewness	-0.093583	-0.1348786
Kurtosis	10.81153	7.1021913

Nifty closing values do not follow a lognormal distribution and the returns do not conform to the normal distribution and hence pricing the returns with the lognormal assumption is inaccurate.

NSE Nifty Frequency plot of daily returns (01.01.2001-01.01.2008)



Methodology

- Optimization done by minimizing $\sum_{i=1}^n (O_i - E_{\Delta}^i)^2$
 O_i is the market price and E_{Δ}^i is the theoretical price using a given set of parameters
- **Hypothesis:** The total error using Weibull distribution is less than that by using Lognormal assumption.
- For comparison, we compute the Error Sum of Squares (ESS) for the two approaches by summing the square of the difference between the predicted and the actual prices.
- These two ESS are then compared for statistically significant difference using the paired 't' test.

Results

t-Test: Paired Two Sample for Means: Combined

	Black-Scholes	Weibull
Mean	8409.500408	6854.498163
Variance	326295883.3	315125699.8
Observations	49	49
Pearson Correlation	0.986598069	
Hypothesized Mean Difference	0	
Df	48	
t Stat	3.692009843	
P(T<=t) one-tail	0.000284234	
t Critical one-tail	1.677224197	
P(T<=t) two-tail	0.000568467	
t Critical two-tail	2.010634722	

Results

t-Test: Paired

**Two Sample for
Means**

25-Oct-2007

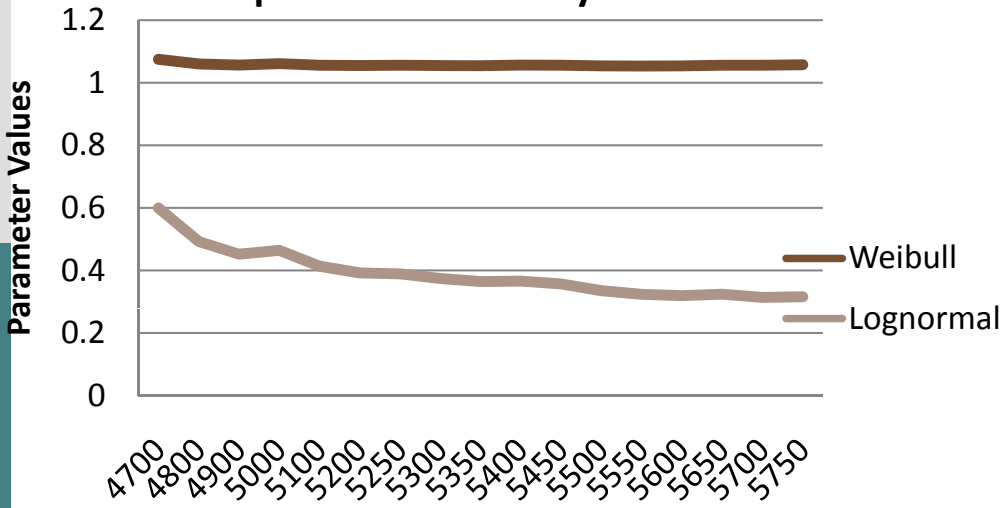
29-Nov-2007

27-Dec-2007

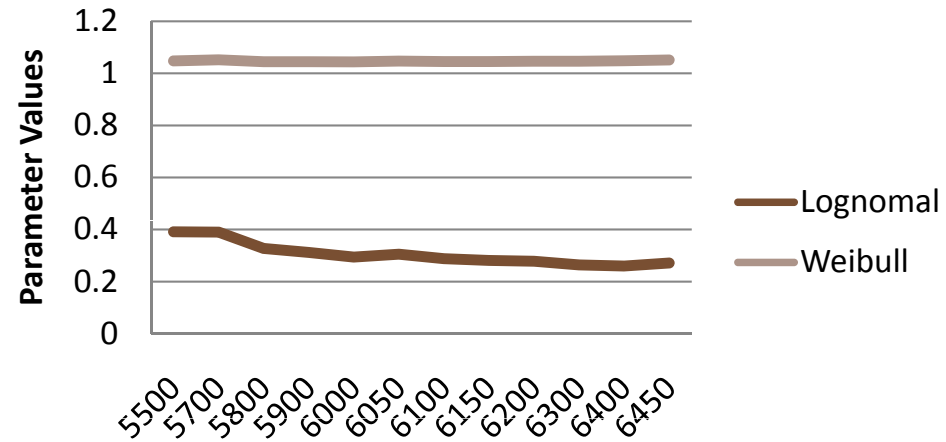
	Black- Scholes	Weibull	Black- Scholes	Weibull	Black- Scholes	Weibull
Mean	7651.2667	5442.9042	13067.94	11482.26	3024.467	2138.164
Variance	45854256	29664205	7.49E+08	7.33E+08	3578593	3871641
Observations	12	12	20	20	15	15
Pearson Correlation	0.9274421		0.989313		0.846583	
Hypothesized Mean Difference	0		0		0	
Df	11		19		14	
t Stat	2.869389		1.77744		3.20391	
P(T<=t) one-tail	0.0076291		0.045753		0.003185	
t Critical one-tail	1.795884		1.72913		1.7613	
P(T<=t) two-tail	0.0152582		0.091507		0.006371	
t Critical two- tail	2.2009852		2.093024		2.144787	

Volatility Smile

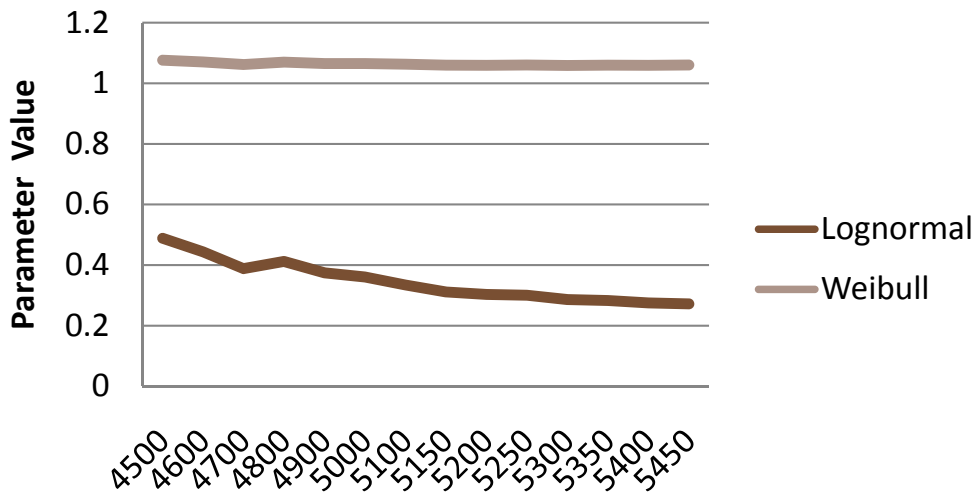
Comparison of volatility smile- Oct 12



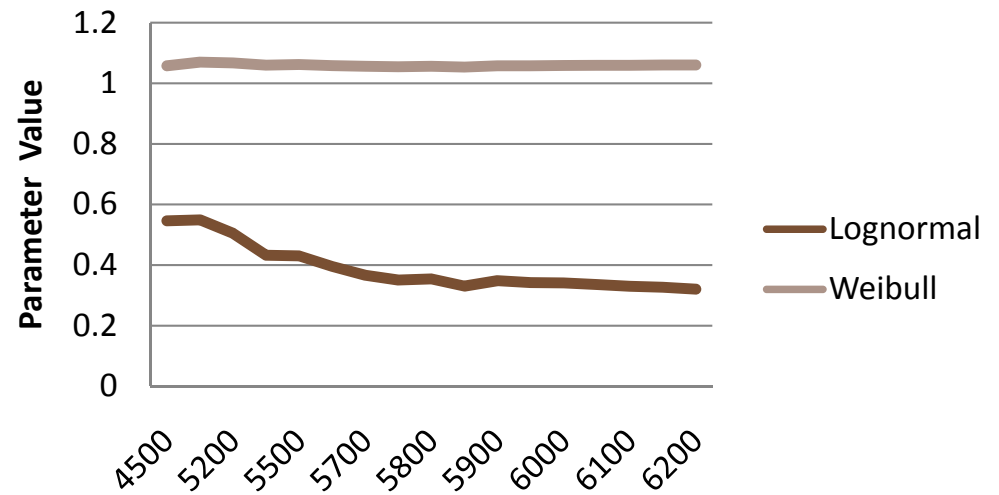
Comparison of volatility smile - december 14



Comparison of volatility smile - Oct 5



Comparison of Volatility smile - Nov 14



Conclusion

The main advantages of the proposed distribution model over the pre-existing models are

- its simple algebraic form, comparable to that of the Black–Scholes model with lognormal distribution,
- the ease of the model's implementation, and
- the absence of pricing biases such as those generated by the standard Black–Scholes formula.

As a result, the proposed model has a potential for a wide range of practical applications.

Future Work

- In this study, we have calculated implied volatility based on “today’s data” to predict “tomorrow’s prices”. This can be extended to explore whether the modified approach gives significantly better prices for longer durations or not.
- A related study can be conducted regarding comparison of different distributional assumptions (g-and-h, GB2, Burr) with the Weibull distribution in pricing of the Nifty options.

Thank you

References

- ❑ *Savickas, R., "A Simple Option Pricing Formula", Department of Finance, George Washington University (2001)*
- ❑ *Black, F. and M. Scholes, 1973. The pricing of options and corporate liabilities, Journal of Political Economy 81, 627–59.*
- ❑ *McDonald, J. B. and R. M. Bookstaber, "Option Pricing for Generalized Distributions." Communications in Statistics - Theory Meth. Vol. 20, 1991.*
- ❑ *Hull, J. C., Options, Futures, & Other Derivatives. Upper Saddle River, NJ: Prentice-Hall, Inc., 2000.*
- ❑ *Data on option prices, Retrieved February 23, 2008 from <http://www.nseindia.com>*
- ❑ *Saurabha, Rritu and Tiwari, Manvendra, Empirical Study of the effect of including Skewness and Kurtosis in Black Scholes option pricing formula on S&P CNX Nifty index Options, IIM Lucknow (2007)*
- ❑ *Merrill, C. and D. F. Babbel, "Interest-Rate Option Pricing Revisited." The Journal of Futures Markets, Vol. 16, No. 8, 1996.*
- ❑ *Misra, D., Kannan, R. and Misra, Sangeeta D. (2006): "Implied Volatility Surfaces: A study of NSE Nifty Options," International Research Journal of Finance & Economics.*
- ❑ *Dutta and Babel, "Extracting Probabilistic Information from the Prices of Interest Rate Options: Tests of Distributional Assumptions", Wharton financial institutions center(2002)*