# *"CHOICE OF INTEREST RATE MODELS FOR ESTIMATING ECONOMIC CAPITAL OF LIFE INSURANCE COMPANIES"*

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## Introduction:

This paper is about choosing an appropriate interest rate model for estimating the economic capital of a life insurance company.

For the purpose of this paper, we will define Economic Capital as the "*Capital that is needed to fund future obligations of a life insurance company with some degree of certainty over a defined time horizon*".

There has been a lot of discussion and debate around the world on estimating economic capital for life insurance companies. Some of the recent developments in this regard include:

- Individual Capital Assessment – United Kingdom
- Swiss Solvency Test – Switzerland
- Solvency II – European Union
- Standard 4360 – Australia/ New Zealand
- C3 Phase II – United States.

These developments have occurred in response to the growing recognition of the fact that the traditional factor-based risk capital calculations cannot be sustained in an insurance market where life insurance products are neither homogeneous nor simple.
In fact it is widely acknowledged that today's insurance markets require capitalization that addresses the risk profile of each life insurance company. However there is no widespread consensus on how this capital [economic capital] needs to be determined.

Broadly the methods for estimating economic capital for insurance companies can be placed under one of the following three categories:

- Fair Value Approach
- Regulatory Solvency Approach
- Cash Balance Approach.

Of these approaches, the fair value approach seems to be gaining ground particularly in Europe. Estimating economic capital using the Fair Value Approach involves simulating multiple interest rate paths using a risk-neutral interest rate model. Hence choosing an appropriate interest rate model turns out to be an important step under this approach.

## Layout of the Paper:

The focus of this paper is on evaluating the available interest rate models in order to home in on an interest rate model that would be appropriate for simulating the interest rate paths under the Fair Value Approach. The paper reflects the empirical work done by the authors in this direction. The paper is structured as follows:

- The first part dwells on the mechanics of the fair value approach in order to set the context for choosing an appropriate risk interest rate model.
- The second part provides a brief overview of the available interest rate models.

- The third part covers the criteria used by the authors for evaluating the available interest rate models from the standpoint of choosing an appropriate model under the Fair Value Approach
- The fourth part provides the concluding thoughts.

## Fair Value Approach: An Over View

Under this approach, economic capital is calculated using a balance sheet approach by valuing both assets and liabilities at their fair values at a horizon point which is typically one year hence. Although the time horizon is one year, the remaining life of both the assets and liabilities needs to be considered for determining the fair value of the assets and the liabilities.

Economic Capital under this approach is defined by examining the distribution of the present value of the economic surplus [defined as fair value of assets less fair value of liabilities] one year hence resulting from simulations across the various risk elements.

The resulting present values of economic surplus when rank -ordered define a distribution and a point in the tail of the distribution is used to define the capital amount. Thus economic capital under this approach is the amount needed today to ensure economic solvency one year hence at a stated probability level [say, a conditional tail expectation {CTE} of 99.5%].

From the above discussion it follows that the fair value approach calls for estimating the economic surplus at T=1..It is important to note that in doing so we need to mark the balance sheet at time T= 0 and time T= 1 year. This gives rise to nested stochastics (scenarios at time T= 0 and another set of stochastic scenarios from each path at time T = 1 year) as shown in the following diagram:

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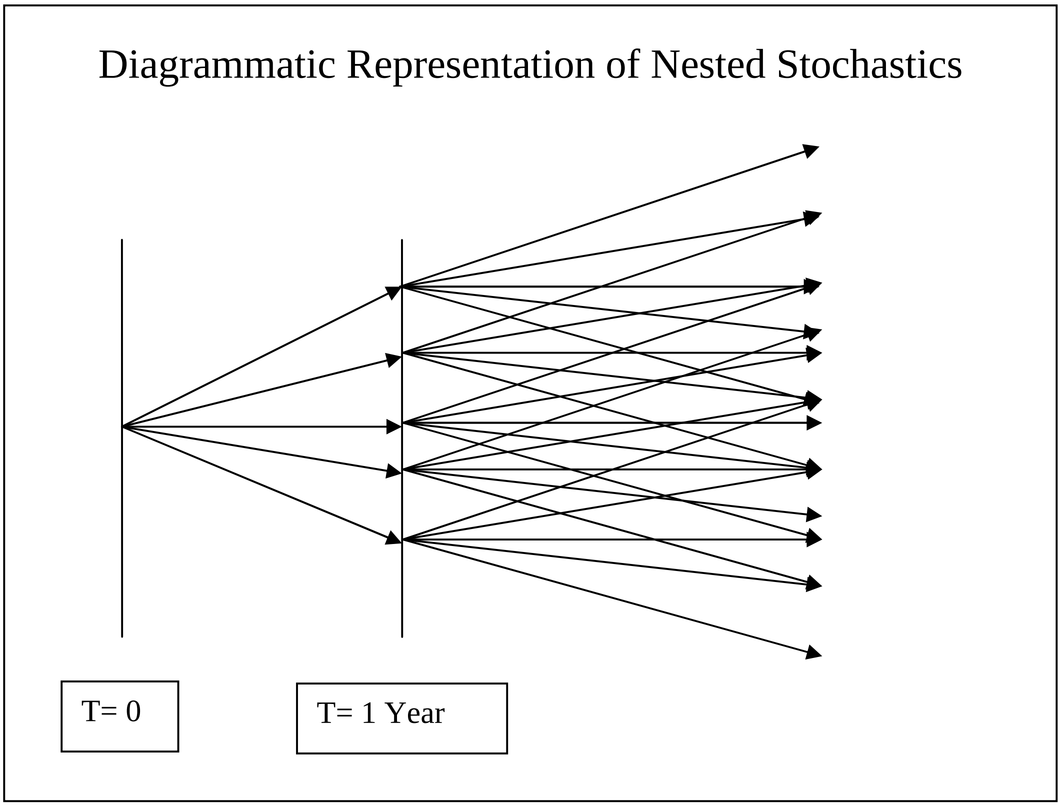


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To perform the stochastic scenarios under the Fair Value Approach, we require interest rate paths to come from a risk neutral interest rate model. It is important to ensure that this interest rate model, interalia, meets the following criteria:
[a] the model is arbitrage-free
[b] the model is consistent with the current yield curve.
[c] the model is well calibrated so as to generate prices for interest rate derivatives [like caps, floors and swaptions] which are consistent with the market prices of these instruments.

## Interest Rate Models: A Brief Overview

Broadly interest rate models can be classified under two categories: [a] Short Rate Models and [b] Whole Yield Curve Models.

- Short Rate Models: These models are typically models of the evolution of the short-term interest rates on the premise that short rate drives the behavior of the term structure of interest rates. The complete yield curve is finally determined by the level of short rate and other model parameters.

  The short rate models are the ones that are commonly used because of the fact that the stochastic evolution of yield curve can be explained to a large extent by the first principal component. The short rate can be a good proxy for first component, also known as 'level' component of the yield curve. Also short rate models have the Markov property meaning that the evolution of the short rate at each instant depends only on its current value- not on how it got there. Another important feature of the popular short rate models like 'Hull and White', and 'Black and Karsinski' models is the deterministic or time-dependent mean reversion feature of these models.

To provide a flavor of the short-rate models we have briefly described the Hull & White one factor model. This model expresses the continuous time evolution of the instantaneous spot rate as:

$\triangle r(t) = (\mu(t) + \alpha\ (\gamma(t) - r(t))\ )\ dt\ + \sigma(t)\ dz(t)$

Where:

$r(t)$ = the short rate at time t

$\mu(t)$ = pure drift term

$\alpha\ (\gamma(t) - r(t)$ = mean reversion term

$\sigma(t)$ = instantaneous volatility of spot interest rates

$dz(t)$ = Standard Wiener Process

The mean reversion term incorporated in this model causes the interest rates to revert to time-varying normal value $\gamma(t)$.

- Whole Yield Curve Models: These models allow the modeler to develop each element of the entire term structure (by evolving a complete set of forward interest rates). These models typically follow the approach of Heath, Jarrow, and Morton (HJM), or the "Market Model" approach of Brace, Gatarek, and Musiella (BGM). By their nature, these models allow for a richer description of the dynamics of the whole term structure.

  The Libor Market Model [LMM] which is a popular model under this category can be represented as follows [assuming no drifts between the reset dates]:

$$F_k(t_{j+1}) = F_k(t_j)\exp\left[\left(\sum_{j=k}^{i}\frac{\delta_i F_i(t_j)\Lambda_{i-j}\Lambda_{k-j}}{1+\delta_j F_i(t_j)} - \frac{\Lambda_{k-j}^2}{2}\right)\delta_k + \Lambda_{k-j}\varepsilon\sqrt{\delta_j}\right]$$

$F_k(t_j)$ : Forward Rate between time $t_k$ and $t_{k+1}$ at time $t_j$

$\delta_i$ : Difference between time $t_k$ and $t_{k+1}$

$\Lambda_i$ : Volatility of $F_K$ for i accrual period between $t_K$ and next reset date

$\varepsilon$. : Random normal variate

In the original specification of the Libor Market Model, a different random variate drives the dynamics of each forward rate, leading to an N-factor model if N forward rates are being simulated. However, the dynamics of neighboring forward rates are in fact highly correlated. Therefore, in practice it is usually possible to reduce the number of stochastic variables used. In this paper we have used 3 independent factors to drive the forward rate dynamics, based on principal component analysis of assumed correlation matrix.

The Volatility structure, which goes as an input to the Libor Market Model, can be determined from market prices of caps and floors. This is considered

to be a major advantage of using Libor Market Model. One can determine the volatility structure directly from the available market prices of caps or floors and then can use that to generate interest rate scenarios. In practice, the accuracy of such calibration depends on number of scenarios generated. In our empirical work that uses antithetic variates, we have observed that 1000 scenarios generate model cap/floor prices that are not significantly different from the market prices of the traded caps and floors.
Similarly we have used some analytical approximations like the Rebonato approximation to define volatility structure beforehand so that model swaption prices are not too far from market prices.

The following diagram illustrates the model inputs for Libor Market Model:

| **Model Inputs** | | |
|---|---|---|
| **From Yield Curve** | **From the Cap/Floor prices** | **From Swaption prices** |
| • Obtain Zero Coupon Curve/ Market Implied Forward Curve<br>• Interpolation, wherever required | • Conversion to Caplet/Floorlet spot volatilities | • Volatilities using analytical approximations |

The LIBOR Market Model [LMM] is designed in such a way that forward rates are log-normally distributed, which is in line with current market practice for quoting cap, floor and swaption prices using the Black formula.

## Performance Criteria for Interest Rate Models

The choice of an appropriate interest rate model will require assessing a given model using the following criteria:

- **No Arbitrage**

  The basic requirement of any Interest Rate model is that it should be able to produce arbitrage free rates. If the model produces arbitrage free rates, then no arbitrage profit can be generated by simple exchange of bonds. The way to ensure no arbitrage is to calibrate the model to the market prices of listed bonds.

- **Mean Reversion**

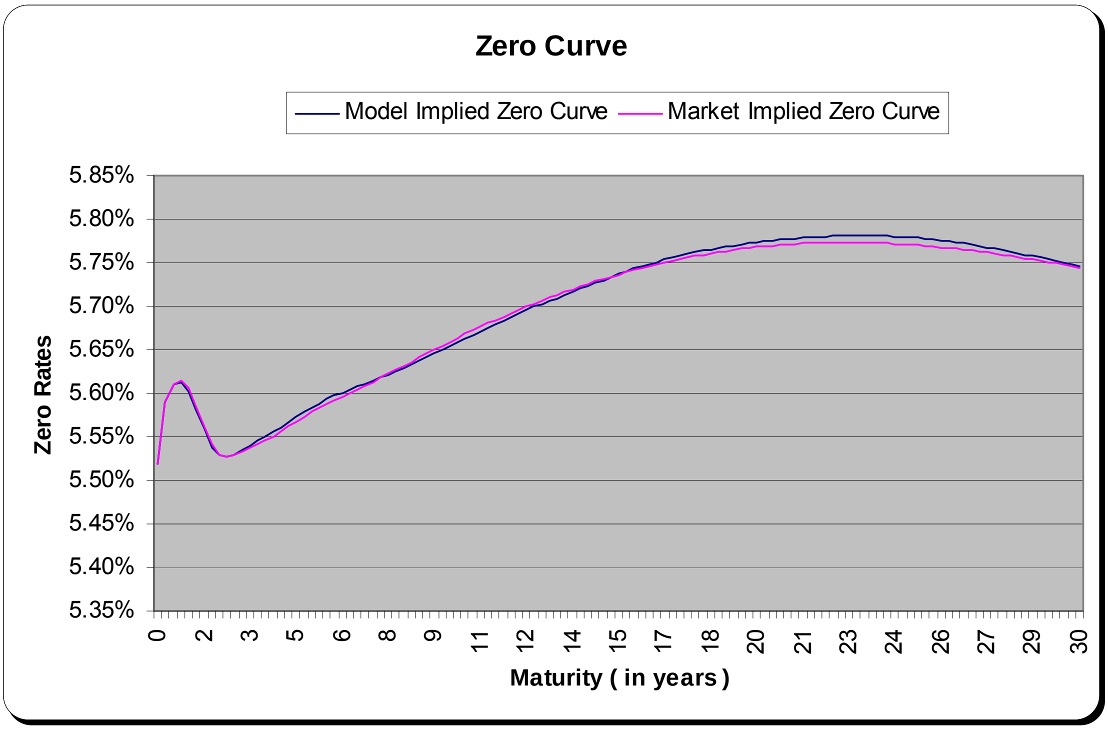
  As per the mean reversion property, interest rates need to revert back to some long-run average level over time. It is a desirable property.

- **Range of Interest rates (Distributional Characteristics)**

  The model equation determines the possible range of rates that model can generate. For example, the Hull and White model described above assumes a normal distribution of rates versus Black and Karsinski assumes a log-normal distribution of interest rates. Thus the Hull and White model has the disadvantage that the short term interest rate can be negative. Black and Karsinski replace the short rate in the Hull and White equation with logarithm of rate. This leaves no possibility for interest rates to become negative. As stated above, Libor Market model allows rates to be distributed log normally and thereby eliminates the possibility of forward rates becoming negative.

- **Calibration to Bond Prices**

  Calibration to bond prices is the basic property that one would expect in the model. Infact, the calibration to bond prices will make the model rates arbitrage-free. Calibration to bond prices simply means that the model's expected value of zero coupon bond prices should match with market implied zero coupon bond prices.
  In case of short rate models, parameters, which may be time dependent, are selected such that the evolution of rates make the zero coupon rates consistent with market implied zero coupon rates. In LMM, calibration to zero coupon rates is performed by construction. The following diagram shows how closely the LMM implied zero rate curve matches with market implied zero rate curve. Clearly the accuracy also depends on the number of scenarios generated. The diagram below is based on 1000 scenarios.




- **Calibration to Interest Rate Derivatives**

As one of the applications of the interest model in the context of life insurance companies will be to value Interest Rate Guarantees embedded in Life Insurance contracts, the ability of the interest rate model to calibrate well to relevant interest rate derivatives becomes a essential feature for the model.

For example, consider a Universal Life product with an embedded interest rate guarantee whereby the insurer will provide at least a stated minimum interest rate on the policy's cash value each year, This guarantee can be viewed as selling an interest rate derivative in the market. This interest rate which is declared by the company from time to time is known as credited rate.
It is typical foe insurance companies selling such contracts to tie the credited rate to the yield on a standard investment portfolio or index. For example the standard portfolio can be a specified basket of government treasury bills. It then becomes important to understand the type of derivative that insurance company is selling which involves understanding the link between the yield on the standard investment portfolio and the market interest rates. For example, the yield on the standard investment portfolio may be approximated by a moving average of prior market interest rates. In that case the guarantee which has been sold by the insurance company is like an Asian Floor* on interest rates with strike price equal to minimum guarantee. Thus it is essential to understand the nature of the

* In the case of a vanilla floor the buyer of the floor receives money if on the maturity of any of the floorlets, the reference rate is below the agreed strike price of the floor. In Asian floors, the reference rate is determined by the average of some pre-determined rate, say LIBOR, over some pre-set period of time.

interest rate guarantee embedded in the life insurance contracts in order to define the exact financial instruments to which the interest rate model needs to be calibrated.

- It is also a known fact that interest rates in the market impact lapse behavior of policyholders. Hence, it becomes important that calibration also captures the lapse behavior. Policyholders have the option to surrender their policy and receive the cash value (less applicable surrender charges) at any time. It is typical that lapse behavior gets activated at higher interest rates because as rates rise, the policyholder will have more of an incentive to receive the cash and reinvest it at a (higher) market rate of return than what they are earning. Policyholder lapsation at this time is also most painful for the insurer, since they may have to liquidate their asset portfolio in a high interest rate environment, thus realizing capital losses. Therefore, allowing a policyholder to surrender is like issuing put options on the same standard investment portfolio mentioned above. It therefore becomes essential that the interest rate model also calibrates well to relevant market traded put options on bonds [at strikes which match the interest rate levels where the policyholder behavior is most activated]. This involves establishing the linkage between lapse behavior and interest rates.

- **Calibration Issues in the Context of Estimating Economic Capital**

It is important to recognize that for estimating economic capital under the fair value approach, interest rate models have to be used at three key steps of the process. **First**, to arrive at fair value of liabilities at T=0. **Second**, to produce interest rate paths between T=0 and T=1 for the simulation of the balance sheet and calculation of the "tail" of the loss distribution (which then determines the economic capital requirement). **Third**, to arrive at fair value of liabilities at T=1.

Because an interest rate model will be used in these three steps, it is natural to examine whether a single calibration is sufficient for use in all three cases, or whether different calibrations (perhaps even different models!) are more appropriate in each case.

Clearly, for the first step [referred to above] , a risk-neutral model is required, which is properly calibrated according to the above discussion. For the second step, one may use the same set of risk neutral scenarios that were used for defining fair value of liabilities at time 0 or one may use "real world" scenarios. By "real world scenarios", we mean a set of scenarios generated according to the "real world" probability measure rather than the "risk neutral" probabilities.
Typically these "real world" probabilities are determined by a fit to historical data, rather than a fit to the current market prices. Whether to use risk neutral or real world scenarios is really the management decision. The advantage that real world scenarios provide over risk neutral scenarios is a better understanding of the economic capital results in terms of associated probability (say 95% CTE). The disadvantage of using "real world" scenarios is the implicit assumption that the probabilities of future events can be derived from past events. This assumption (which has often proven spectacularly false!) more often than not lead companies to a false reading on the true level of risk embedded in their positions. The Long Term Capital Management [LTCM} case study is an excellent example of this.

For the third step, a risk-neutral calibrated model is again required, because the balance sheet needs to be marked -to-market at T=1. There is an additional question of exactly how to calibrate this model, since the prices of the financial instruments [to which the model was calibrated at T=0] could have changed by virtue of the simulation along scenarios produced in step 2.
Coming to the original question on choice of model for calibration, LMM [Libor Market Model] provides some definitive advantages. The following table shows a single scenario for Libor Market model. It also tries to show the rates that get used in calculation of caps, floors and swaptions.

Scenario Id

Time steps

Enough to calculate Cap/Floor prices

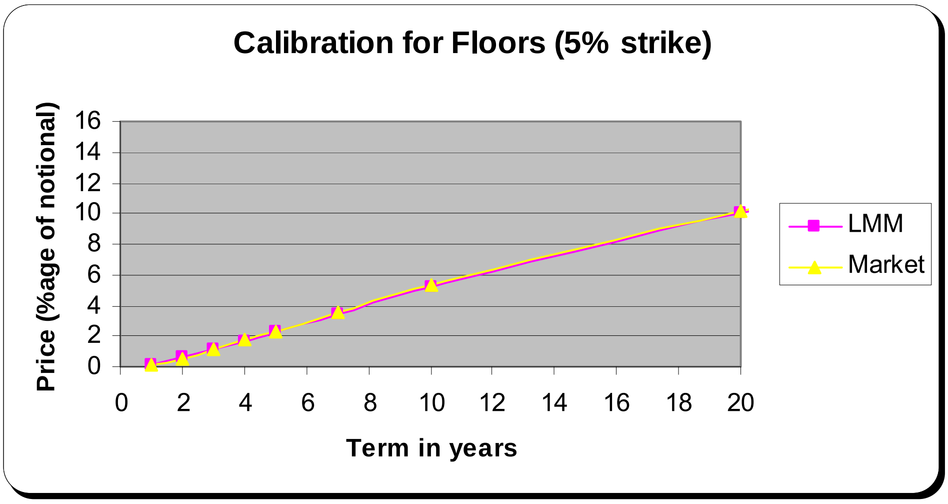
| | | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 0.054162 | Expired | Expired | Expired | Expired | Expired | Expired | Expired | Expired | Expired | Expired | Expired | Expired |
| 1 | 1 | 0.053424 | 0.065099 | Expired | Expired | Expired | Expired | Expired | Expired | Expired | Expired | Expired | Expired | Expired |
| 1 | 2 | 0.051833 | 0.06333 | 0.060793 | Expired | Expired | Expired | Expired | Expired | Expired | Expired | Expired | Expired | Expired |
| 1 | 3 | 0.05011 | 0.061365 | 0.058879 | 0.057376 | Expired | Expired | Expired | Expired | Expired | Expired | Expired | Expired | Expired |
| 1 | 4 | 0.049042 | 0.060173 | 0.057713 | 0.056229 | 0.052597 | Expired | Expired | Expired | Expired | Expired | Expired | Expired | Expired |
| 1 | 5 | 0.048404 | 0.059485 | 0.057036 | 0.05555 | 0.051922 | 0.049107 | Expired | Expired | Expired | Expired | Expired | Expired | Expired |
| 1 | 6 | 0.048074 | 0.059155 | 0.056707 | 0.055218 | 0.051573 | 0.048745 | 0.052413 | Expired | Expired | Expired | Expired | Expired | Expired |
| 1 | 7 | 0.048028 | 0.059157 | 0.056701 | 0.055205 | 0.051527 | 0.048676 | 0.05239 | 0.052265 | Expired | Expired | Expired | Expired | Expired |
| 1 | 8 | 0.048227 | 0.059445 | 0.056973 | 0.055464 | 0.051741 | 0.048859 | 0.05263 | 0.052504 | 0.058087 | Expired | Expired | Expired | Expired |
| 1 | 9 | 0.048563 | 0.059887 | 0.057396 | 0.055871 | 0.052099 | 0.049183 | 0.053016 | 0.052888 | 0.058595 | 0.054046 | Expired | Expired | Expired |
| 1 | 10 | 0.04895 | 0.060379 | 0.057871 | 0.056332 | 0.052511 | 0.049564 | 0.053456 | 0.053327 | 0.059154 | 0.054504 | 0.053928 | Expired | Expired |
| 1 | 11 | 0.049321 | 0.060838 | 0.058316 | 0.056767 | 0.052902 | 0.04993 | 0.053874 | 0.053745 | 0.05968 | 0.05494 | 0.054356 | 0.061457 | Expired |
| 1 | 12 | 0.04963 | 0.061209 | 0.058681 | 0.057124 | 0.053226 | 0.050237 | 0.054223 | 0.054092 | 0.06012 | 0.055304 | 0.054715 | 0.061965 | 0.065097 |

Forward Rate Curve at t=0

Forward Rate Curve at t=4 (Required to calculate swaption prices at t=4)

The important point to note here is that short rate models don't give such a rich description of yield curves at future time steps. In the case of short rate models one needs to rely either on analytical solution or nested stochastics to come up with implied forward curve at each time step for each scenario. Also ,in the case of short rate models there is a need to do an optimization exercise in order to arrive at the model parameters like volatility [in order to ensure that the prices of the interest rate derivatives derived from the model is consistent with their corresponding market prices] . This can turn out to be a computationally intensive exercise particularly when an analytical solution is not available.

On the other hand, we can estimate the volatility structure out of the Libor Market Model [LMM] and can generate scenarios that calibrates well to floors and caps. Hence, it can be said that LMM calibrates caplets and floorlets by construction. Below are the calibration results for floors using 1000 scenarios based on the LMM.




- **Volatility of Long Term Rates**

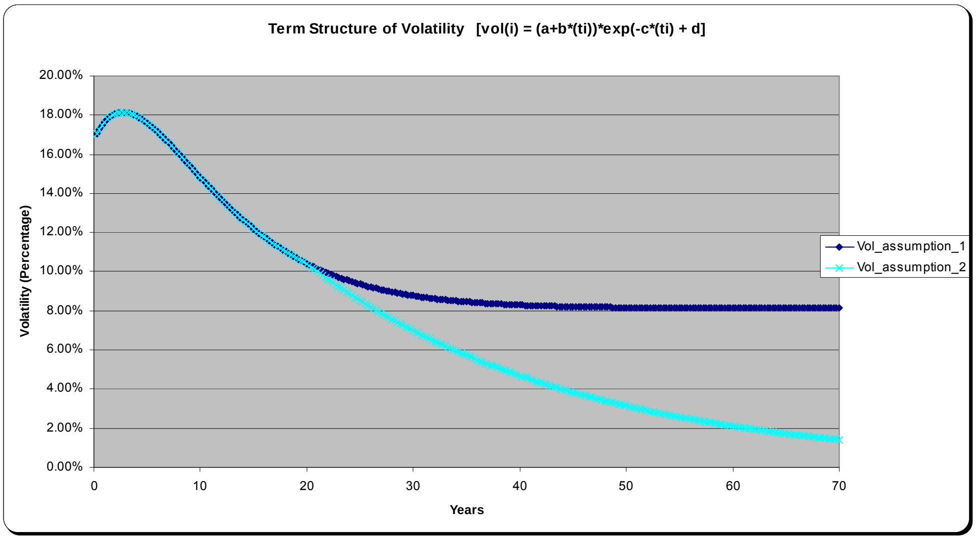
As the term to maturity of life insurance contracts is fairly long, the assumption about volatility of long term interest rates becomes very important. An obvious example is the Universal Life contract where the policy term typically exceeds the term to maturity of long dated financial instruments available in the capital market.

The assumption about interest rate volatility for longer terms [particularly for the period beyond the term of long dated financial instruments] can have a significant impact on the generation of scenarios and hence on estimating economic capital.

For example, consider the following parametric form for volatility structure up to year 20 [assuming that the term of the longest financial instrument in the market is 20 years] :

Vol (i) = (a+b*(ti))*exp (-c*(ti) + d.

Beyond year 20, we need to make an assumption about the volatility structure. We can make an assumption that beyond year 20 there will be no change in the above parametric form. On the other hand we can assume that the above parametric form will be subject to a decay factor beyond year 20. .The following diagram provides a graphic representation of these two assumptions:




Our empirical work reveals that this assumption about volatility structure has a significant impact on the generation of scenarios. We observed that the assumption of no change in the parametric form beyond year 20 leads to generation of significant number of run away scenarios† particularly over the longer terms [beyond 40 years]. This was not the case with the second assumption where the parametric form was subject to a decay factor beyond year 20.

## Concluding Remarks

This paper highlights how the choice of interest rate models can be an intensive exercise in estimating the economic capital under the fair value approach. It also provides some insights into the evaluation criteria that one needs to consider before choosing an interest rate model for estimating economic capital. Our empirical work suggests that a Whole Yield Curve Model [like the Libor Market Model] meets these evaluation criteria better than the short rate models.

Clearly choosing an appropriate interest rate model is just the first step of estimating economic capital. Ultimately, the estimation of economic capital is a multiple step process and depends on many assumptions beyond interest rate models. The choice of interest rate model, though the first step is not independent of other steps. It is important to ensure that it is consistent with all the steps and assumptions that one needs to make while estimating economic capital.

† We have defined run away scenarios as the ones that generate rates of 200% and more. In long term, we observed our model (with volatility assumption 1) generating scenarios with rates as high as 1000%.

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