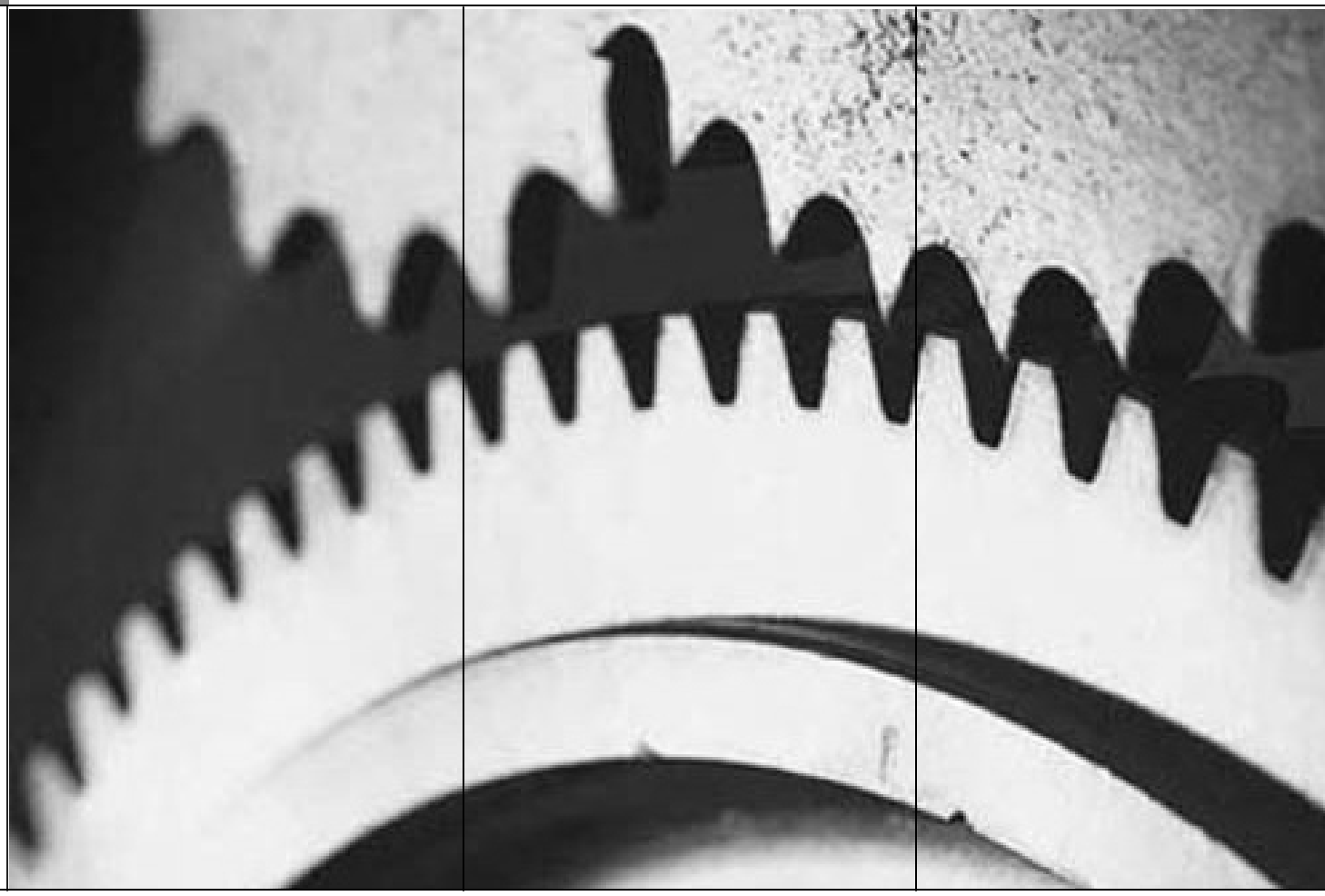
# 6th GLOBAL CONFERENCE OF ACTUARIES
## 18 – 19 February, 2004, New Delhi

**By Walter de Oude**

**(Subject Code 01 – Subject Group : Life Insurance)**

Swiss Re

# Cashflow pricing techniques



Cashflow Pricing
Walter de Oude
5th GCA Feb 2004

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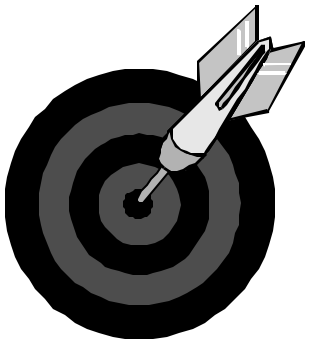
# Objectives



- Illustrate the advantages of cash flow methods over traditional methods
- Give practical examples of how cash flow methods can be used for pricing
- Outline some useful techniques, based on the cash flow approach, which will enable us to better understand the business that we write

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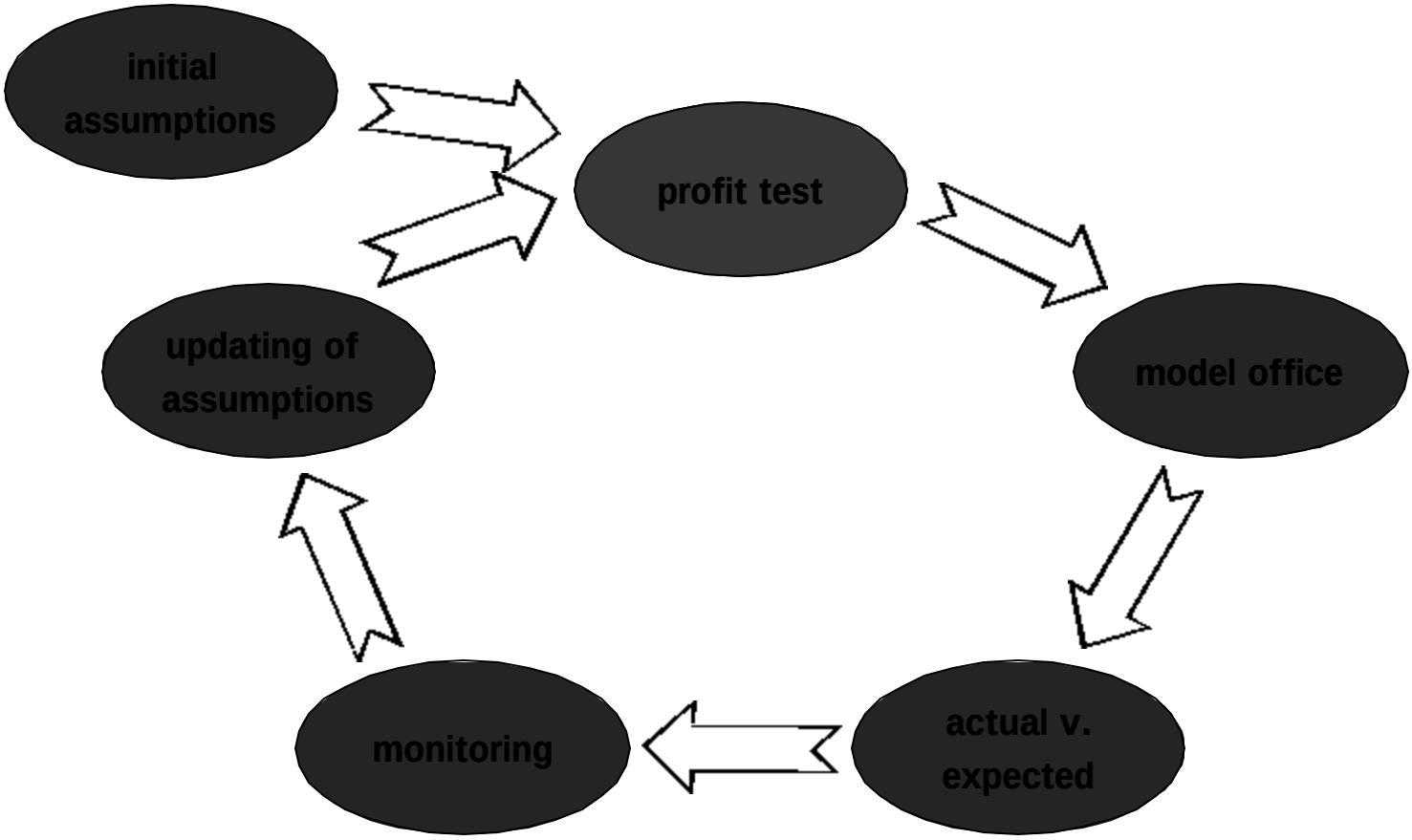
# The Actuarial Control Cycle



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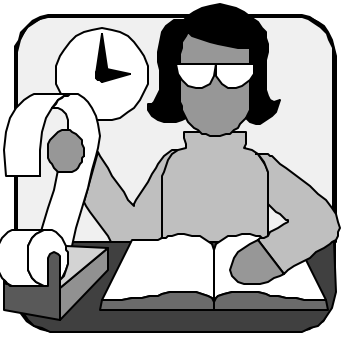
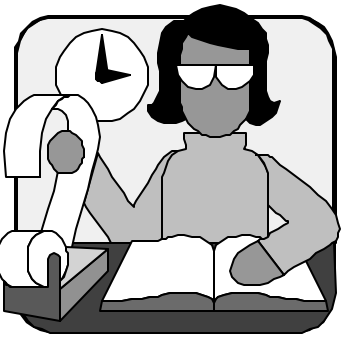
# Traditional Methods (1)



- Developed before computers were widely available
- Use tables of annuity ($\ddot{a}_x$) and assurance ($A_x$) factors, and commutation functions ($C_x$, $D_x$, $N_x$, $M_x$, $R_x$, etc)

# Traditional Methods (2)

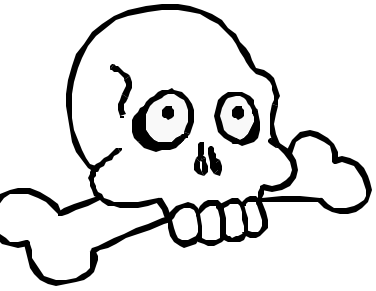


- Determining premiums involved solving an equation like:

$$P\ddot{a}_{x:\overline{n}|} = SA_{x:\overline{n}|} + I + mP\ddot{a}_{x:\overline{n}|}$$

- where
  - P, S = premium, sum insured
  - I, m =initial, maintenance expenses

# Problems with Traditional Methods



- Assumptions are inflexible (eg interest rate cannot be varied over time)
- Difficult to allow for complex benefits (eg return of premiums on death)
- Can only allow for one decrement
  (eg no allowance for profits and losses on surrenders, difficult to include rider benefits)

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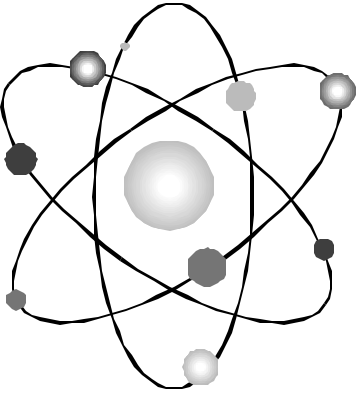
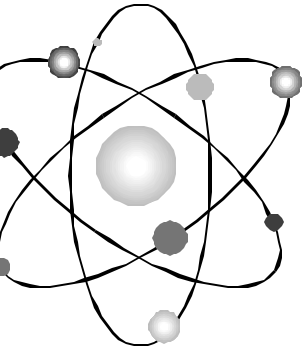
# Cash Flow Methods (1)



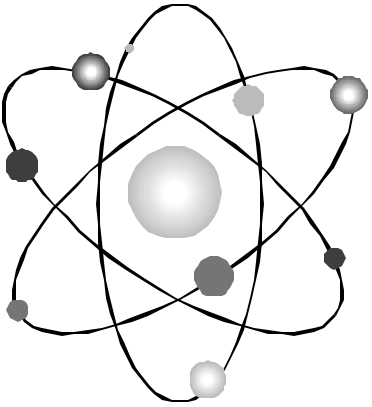
- Make use of computers for complex calculations
- More complicated models are possible allowing us to overcome these disadvantages
- More intuitive and easier to understand for non-actuaries

# Cash Flow Methods (2)



- Can incorporate assumptions about:
  - surrenders and lapses **4**
  - riders and additional benefits **4**
  - valuation basis and method **4**
  - new business **4**
  - profitability targets **4**
  - cost of capital
  - bonuses and distribution of surplus
  - reinsurance

# Cash Flow Methods (3)



- The basic idea
  - Set up a column for *every* cash flow relevant to the policy, and some columns for intermediate calculations if required
  - For each column, multiply the *possible* cash flow by the *probability* of that cash flow occurring to get an *expected* cash flow
  - Manipulate the cash flows to arrive at the desired result

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# Examples - Products

- Endowment Insurance
- Term Insurance
- It is very easy to adapt cash flow pricing techniques to other products (eg annuities, unit-linked, etc)

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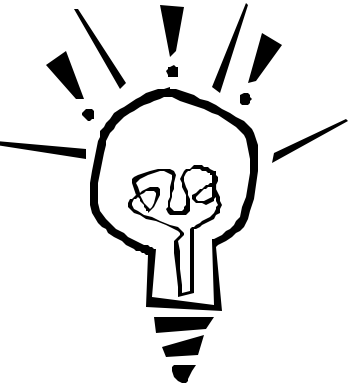
# Examples - Techniques

- Premium Rating
- Measures of Profitability
- Sensitivity and Scenario Testing
- Surrender Values
- Adding a Rider
- Model Office
- New Business Projections and Capital Requirements

# A Simple Example



- 10 year endowment policy
- 45 year-old male
- non-profit
- Sum Insured = Rs.10,000
- Mortality = A67/70 Ult
- 4% Interest
- No expenses

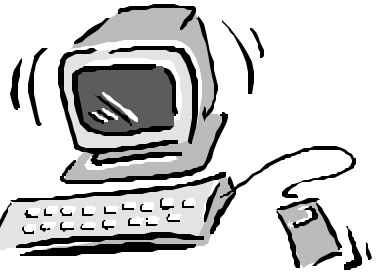
# Premium Rating (1)

- Traditional way of setting premium:

$$P\ddot{a}_{45:\overline{10}|} = 10{,}000 A_{45:\overline{10}|}$$

- Solve for P by looking up tables
  P = Rs. 819.35

# Premium Rating (2)



- The Cash Flow Pricing Approach
  - Set up columns of cash flows (Premiums, Death & Maturity Claims, Interest, Accumulated Cash Flow)
  - Guess a reasonable premium
  - Use "Goal Seek" or "Solver" to find the premium which makes the accumulated cash flow equal to ZERO at the end of the policy term

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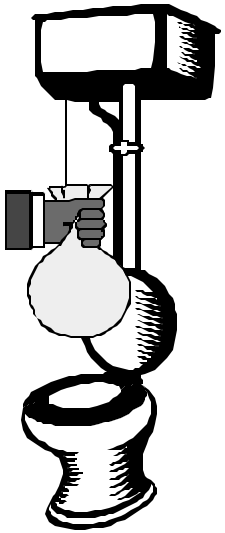
# Premium Rating (3)

- The Result:

| Projection | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | | | | | | | |
| Year | Age | lx | qx | Premium | Death Claims | Maturity Claims | Interest | Accumulated Cash Flow |
| | | | | BOY | EOY | EOY | EOY | EOY |
| 1 | 45 | 1.00000 | 0.00264 | **819.35** | 26.37 | 0.00 | 32.77 | 825.75 |
| 2 | 46 | 0.99736 | 0.00298 | 817.19 | 29.69 | 0.00 | 65.72 | 1678.96 |
| 3 | 47 | 0.99439 | 0.00336 | 814.76 | 33.39 | 0.00 | 99.75 | 2560.07 |
| 4 | 48 | 0.99105 | 0.00378 | 812.02 | 37.50 | 0.00 | 134.88 | 3469.47 |
| 5 | 49 | 0.98730 | 0.00426 | 808.95 | 42.05 | 0.00 | 171.14 | 4407.51 |
| 6 | 50 | 0.98310 | 0.00479 | 805.50 | 47.08 | 0.00 | 208.52 | 5374.44 |
| 7 | 51 | 0.97839 | 0.00538 | 801.65 | 52.61 | 0.00 | 247.04 | 6370.52 |
| 8 | 52 | 0.97313 | 0.00603 | 797.33 | 58.69 | 0.00 | 286.71 | 7395.88 |
| 9 | 53 | 0.96726 | 0.00676 | 792.53 | 65.34 | 0.00 | 327.54 | 8450.60 |
| 10 | 54 | 0.96073 | 0.00756 | 787.17 | 72.59 | 9534.69 | 369.51 | 0.00 |

# Premium Rating - Expenses (1)



- Initial Expenses: Rs.80 per policy
- Maintenance Expenses:
  Rs.10 per policy (including 1st year)
- New Equation:

$$P\ddot{a}_{45:\overline{10}|} = 10{,}000\, A_{45:\overline{10}|} + 80 + 10\ddot{a}_{45:\overline{10}|}$$

- P = Rs.838.98

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# Premium Rating - Expenses (2)

- The Cash Flow Pricing equivalent:

| Projection | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Year | Age | lx | qx | Premium | Initial Expenses | Maint Expenses | Death Claims | Maturity Claims | Interest | Accumulated Cash Flow |
| | | | | BOY | BOY | BOY | EOY | EOY | EOY | EOY |
| 1 | 45 | 1.00000 | 0.00264 | **838.98** | 80.00 | 10.00 | 26.37 | 0.00 | 29.96 | 752.57 |
| 2 | 46 | 0.99736 | 0.00298 | 836.77 | 0.00 | 9.97 | 29.69 | 0.00 | 63.17 | 1612.84 |
| 3 | 47 | 0.99439 | 0.00336 | 834.28 | 0.00 | 9.94 | 33.39 | 0.00 | 97.49 | 2501.27 |
| 4 | 48 | 0.99105 | 0.00378 | 831.48 | 0.00 | 9.91 | 37.50 | 0.00 | 132.91 | 3418.24 |
| 5 | 49 | 0.98730 | 0.00426 | 828.33 | 0.00 | 9.87 | 42.05 | 0.00 | 169.47 | 4364.12 |
| 6 | 50 | 0.98310 | 0.00479 | 824.80 | 0.00 | 9.83 | 47.08 | 0.00 | 207.16 | 5339.17 |
| 7 | 51 | 0.97839 | 0.00538 | 820.85 | 0.00 | 9.78 | 52.61 | 0.00 | 246.01 | 6343.64 |
| 8 | 52 | 0.97313 | 0.00603 | 816.44 | 0.00 | 9.73 | 58.69 | 0.00 | 286.01 | 7377.66 |
| 9 | 53 | 0.96726 | 0.00676 | 811.51 | 0.00 | 9.67 | 65.34 | 0.00 | 327.18 | 8441.35 |
| 10 | 54 | 0.96073 | 0.00756 | 806.03 | 0.00 | 9.61 | 72.59 | 9534.69 | 369.51 | 0.00 |

# Premium Rating - Reserves (1)

- The Traditional Method implicitly assumes that:
  Reserving Assumptions =
  Pricing Assumptions
  (Reserves = Accumulated Cash Flow)

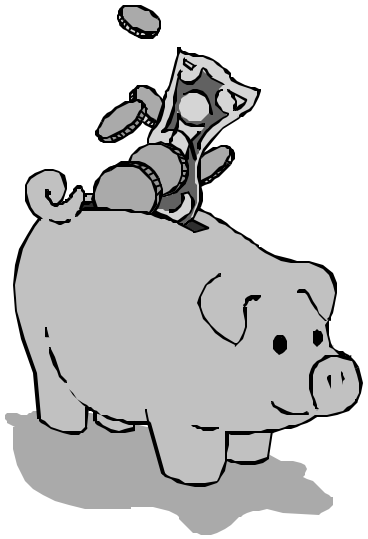
- What if our reserving basis is different to (more conservative than) our pricing basis?

- Or if we have to allocate capital and charge for it separately?

# Premium Rating - Reserves (2)

- What happens now?
  - We no longer have an Accumulated Cash Flow - instead, we have Reserves
  - We need to set up Reserves at each valuation date (assume this happens just before a policy anniversary) for the expected number of policies at that date
  - We need to adjust the cash flows to hold the correct reserve at each valuation date - we may have insufficient or excess cash

# The Concept of Transfers (Surplus) (1)

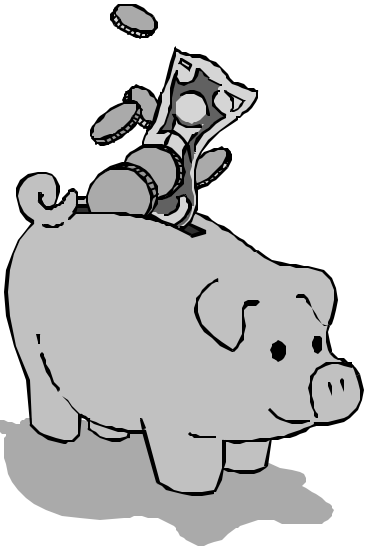


- The Transfer (Surplus) is the remainder of the cash flow after the reserves have been set up for the year
- In General:

  Transfer =

  + Cash Inflows (premiums, interest, etc)
  - Cash Outflows (expenses, claims, etc)
  + Previous Year's Reserves
  - This Year's Reserves

# The Concept of Transfers (Surplus) (2)



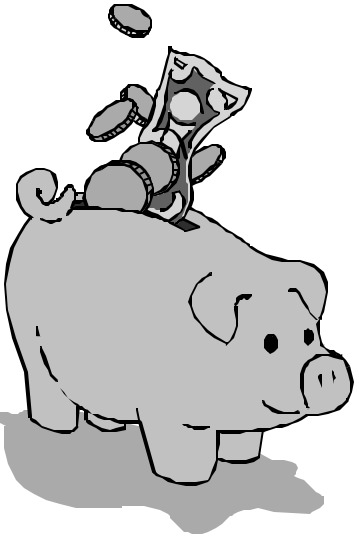
- Back to our example:
  If Reserves are set equal to the Accumulated Cash Flow
  (ie reserving basis = pricing basis)…
  - A67/70 mortality
  - 4% interest
  - 80 Zillmer adjustment for initial expenses

# The Concept of Transfers (Surplus) (3)

- All transfers are equal to zero (ie nothing is left over after the reserves are set up)

| Projection | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Year | Age | lx | qx | Premium | Initial Expenses | Maint Expenses | Death Claims | Maturity Claims | Interest | **Reserves** | **Transfer** |
| | | | | BOY | BOY | BOY | EOY | EOY | EOY | **EOY** | **EOY** |
| 1 | 45 | 1.00000 | 0.00264 | **838.98** | 80.00 | 10.00 | 26.37 | 0.00 | 29.96 | **752.57** | **0.00** |
| 2 | 46 | 0.99736 | 0.00298 | 836.77 | 0.00 | 9.97 | 29.69 | 0.00 | 63.17 | **1612.84** | **0.00** |
| 3 | 47 | 0.99439 | 0.00336 | 834.28 | 0.00 | 9.94 | 33.39 | 0.00 | 97.49 | **2501.27** | **0.00** |
| 4 | 48 | 0.99105 | 0.00378 | 831.48 | 0.00 | 9.91 | 37.50 | 0.00 | 132.91 | **3418.24** | **0.00** |
| 5 | 49 | 0.98730 | 0.00426 | 828.33 | 0.00 | 9.87 | 42.05 | 0.00 | 169.47 | **4364.12** | **0.00** |
| 6 | 50 | 0.98310 | 0.00479 | 824.80 | 0.00 | 9.83 | 47.08 | 0.00 | 207.16 | **5339.17** | **0.00** |
| 7 | 51 | 0.97839 | 0.00538 | 820.85 | 0.00 | 9.78 | 52.61 | 0.00 | 246.01 | **6343.64** | **0.00** |
| 8 | 52 | 0.97313 | 0.00603 | 816.44 | 0.00 | 9.73 | 58.69 | 0.00 | 286.01 | **7377.66** | **0.00** |
| 9 | 53 | 0.96726 | 0.00676 | 811.51 | 0.00 | 9.67 | 65.34 | 0.00 | 327.18 | **8441.35** | **0.00** |
| 10 | 54 | 0.96073 | 0.00756 | 806.03 | 0.00 | 9.61 | 72.59 | 9534.69 | 369.51 | **0.00** | **0.00** |

# The Concept of Transfers (Surplus) (4)



- Now we make our reserving basis more conservative than our pricing basis:
  - A67/70 mortality (same as pricing basis)
  - 3% interest (instead of 4%)
  - 40 Zillmer adjustment for initial expenses (instead of 80)

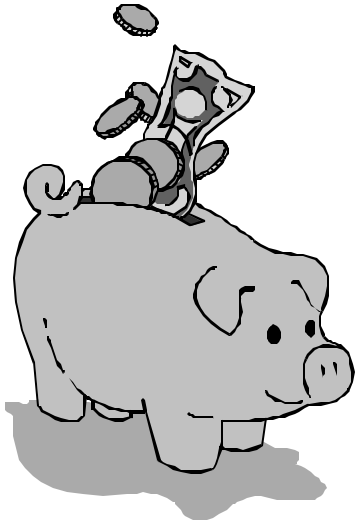
# The Concept of Transfers (Surplus) (5)

- Our pattern of transfers changes
  - negative at the start
  - positive at the end

**Projection**

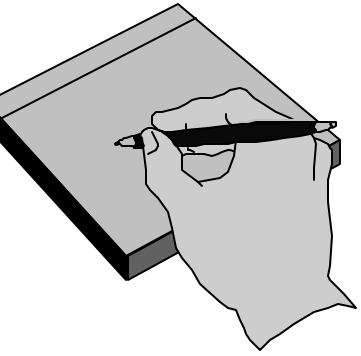
| Year | Age | lx | qx | Premium | Initial Expenses | Maint Expenses | Death Claims | Maturity Claims | Interest | Reserves | **Transfer** |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | BOY | BOY | BOY | EOY | EOY | EOY | EOY | **EOY** |
| 1 | 45 | 1.00000 | 0.00264 | **838.98** | 80.00 | 10.00 | 26.37 | 0.00 | 29.96 | 828.17 | **-75.61** |
| 2 | 46 | 0.99736 | 0.00298 | 836.77 | 0.00 | 9.97 | 29.69 | 0.00 | 66.20 | 1716.71 | **-25.24** |
| 3 | 47 | 0.99439 | 0.00336 | 834.28 | 0.00 | 9.94 | 33.39 | 0.00 | 101.64 | 2625.54 | **-16.25** |
| 4 | 48 | 0.99105 | 0.00378 | 831.48 | 0.00 | 9.91 | 37.50 | 0.00 | 137.88 | 3554.54 | **-7.05** |
| 5 | 49 | 0.98730 | 0.00426 | 828.33 | 0.00 | 9.87 | 42.05 | 0.00 | 174.92 | 4503.50 | **2.37** |
| 6 | 50 | 0.98310 | 0.00479 | 824.80 | 0.00 | 9.83 | 47.08 | 0.00 | 212.74 | 5472.13 | **12.00** |
| 7 | 51 | 0.97839 | 0.00538 | 820.85 | 0.00 | 9.78 | 52.61 | 0.00 | 251.33 | 6460.08 | **21.84** |
| 8 | 52 | 0.97313 | 0.00603 | 816.44 | 0.00 | 9.73 | 58.69 | 0.00 | 290.67 | 7466.87 | **31.90** |
| 9 | 53 | 0.96726 | 0.00676 | 811.51 | 0.00 | 9.67 | 65.34 | 0.00 | 330.75 | 8491.95 | **42.16** |
| 10 | 54 | 0.96073 | 0.00756 | 806.03 | 0.00 | 9.61 | 72.59 | 9534.69 | 371.54 | 0.00 | **52.63** |

# The Concept of Transfers (Surplus) (6)

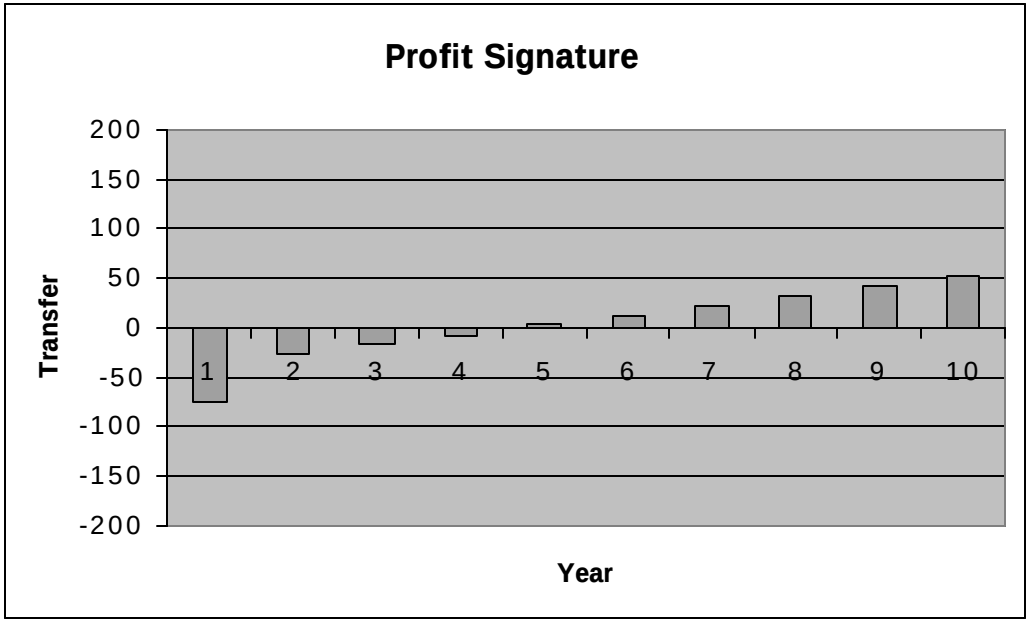


- Why??
- Our reserves are conservative
  - Our cash flows are inadequate to set up the reserves at the beginning - we need to inject capital (negative transfers)
  - We recoup this capital over the term of the policy as we expect experience to be better than the reserving basis (positive transfers)

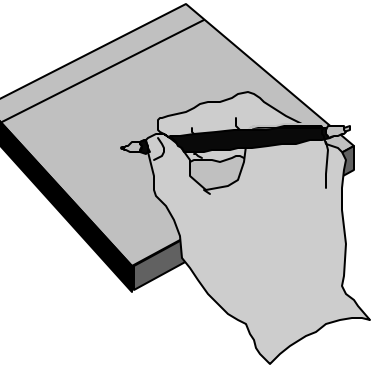
# The Profit Signature (1)



- The pattern or shape of transfers over time - showing the amount of capital invested and its recovery over time



# The Profit Signature (2)



- Ideally, the valuation basis should result in the following pattern of transfers
  - absolute amount not too large (reserving basis not too strong)
  - negative then positive
  - should not turn negative in later years (reserving basis not too weak)

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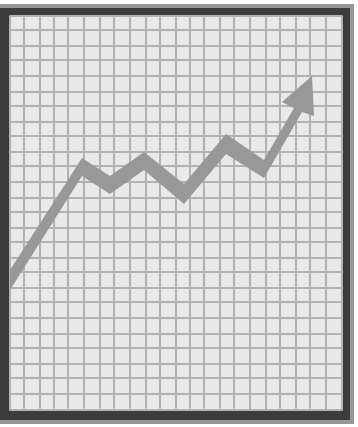
# Why calculate transfers?

- Now that we have injections and releases of capital (transfers), what do we use them for?
  - ANSWER: Measuring Profit !!!
    (NB: these are *expected* profits)

# Measuring Profit (1)

- How did we measure profit using traditional actuarial methods?
  - ANSWER: We didn't
    - The traditional actuarial approach was to build in some implicit conservatism in the premium rates so that we expected a profit
    - Expected profit was never quantified !!!

# Measuring Profit (2)



- Ideally, a measure of profit should consider
  - all cash flows
  - time value of money (ie interest)
  - tax

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# Measuring Profit (3)

- Measures of Profit
  - Payback Period
  - Internal Rate of Return
  - Net Present Value
  - Cost of Capital Comparison

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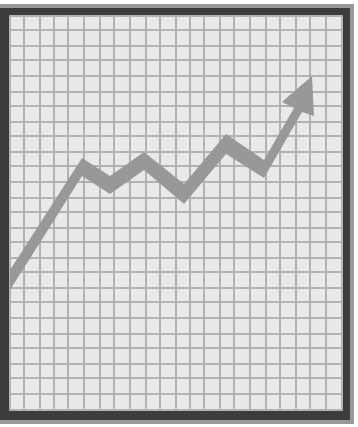
# Payback Period

- Expected number of years to recover original cash investment
- Problems
  - ignores cash flows after the payback period
  - ignores the rate of return required by shareholders
- Not a good measure of profitability

# Internal Rate of Return (IRR)

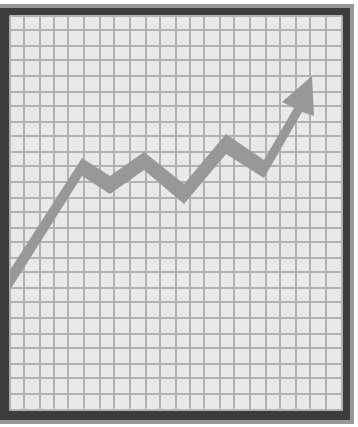


- The discount rate which makes the expected net present value of transfers equal to zero

$$\text{IRR} = i \quad \text{where} \sum_t (1+i)^{-t} \text{Transfer}_t = 0$$

- Problems
  - multiple solutions possible
  - reinvestment assumption

# Net Present Value (NPV) (1)



$$\mathrm{NPV} = \sum_t (1+i)^{-t}\ \mathrm{Transfer}_t$$

- Which discount rate?
  - rate earned on investments
  - shareholders' required rate (risk rate)
- Transfers involve the use of shareholder capital - therefore we should use the shareholders' required rate of return

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# Net Present Value (NPV) (2)

- The expected NPV (discounting at either the earned rate or the required shareholder rate) is sometimes expressed as a percentage of premium (or present value of premiums) for *scaling purposes*

  ie the NPV per unit of premium

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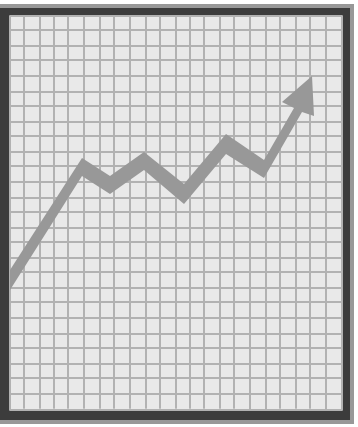
# Cost of Capital



- Measure the return achieved compared with the capital allocated to that policy business. Amount of capital allocated usually based on a combination of:
  - Regulatory capital
  - Respectability capital
  - Risk based capital
- Return required on that capital set by
  - Shareholder requirements
  - Market pressure (returns avaialable elsewhere, etc.)
- Calculated as a deduction in the cashflow model, and compared with the profit signature NPV discounted at an earned rate.

# What does it all mean?

- The IRR can be thought of as the return expected to be earned on the policy
- If the IRR is greater than the risk rate required by shareholders, then shareholders should be happy
- The NPV (discounted at the shareholder risk rate) can be thought of as the expected contribution to shareholder wealth
  - If the NPV is greater than zero, shareholders should be happy

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# Applying the Techniques (1)

- Back to our Example
  - Investment Earning Rate: 4%
  - Shareholders' Risk Rate: 10%

| Measures of Profitability | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| NPV @ earned rate | | | 0.01 | | | | | | | | |
| NPV @ risk rate | | | -34.11 | | | | | | | | |
| IRR | | | 4.00% | | | | | | | | |
| Projection | | | | | | | | | | | |
| Year | Age | lx | qx | Premium | Initial Expenses | Maint Expenses | Death Claims | Maturity Claims | Interest | Reserves | Transfer |
| | | | | BOY | BOY | BOY | EOY | EOY | EOY | EOY | EOY |
| 1 | 45 | 1.00000 | 0.00264 | **838.98** | 80.00 | 10.00 | 26.37 | 0.00 | 29.96 | 828.17 | -75.61 |
| 2 | 46 | 0.99736 | 0.00298 | 836.77 | 0.00 | 9.97 | 29.69 | 0.00 | 66.20 | 1716.71 | -25.24 |
| 3 | 47 | 0.99439 | 0.00336 | 834.28 | 0.00 | 9.94 | 33.39 | 0.00 | 101.64 | 2625.54 | -16.25 |
| 4 | 48 | 0.99105 | 0.00378 | 831.48 | 0.00 | 9.91 | 37.50 | 0.00 | 137.88 | 3554.54 | -7.05 |
| 5 | 49 | 0.98730 | 0.00426 | 828.33 | 0.00 | 9.87 | 42.05 | 0.00 | 174.92 | 4503.50 | 2.37 |
| 6 | 50 | 0.98310 | 0.00479 | 824.80 | 0.00 | 9.83 | 47.08 | 0.00 | 212.74 | 5472.13 | 12.00 |
| 7 | 51 | 0.97839 | 0.00538 | 820.85 | 0.00 | 9.78 | 52.61 | 0.00 | 251.33 | 6460.08 | 21.84 |
| 8 | 52 | 0.97313 | 0.00603 | 816.44 | 0.00 | 9.73 | 58.69 | 0.00 | 290.67 | 7466.87 | 31.90 |
| 9 | 53 | 0.96726 | 0.00676 | 811.51 | 0.00 | 9.67 | 65.34 | 0.00 | 330.75 | 8491.95 | 42.16 |
| 10 | 54 | 0.96073 | 0.00756 | 806.03 | 0.00 | 9.61 | 72.59 | 9534.69 | 371.54 | 0.00 | 52.63 |

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# Applying the Techniques (2)

| Measures of Profitability | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| NPV @ earned rate | | | | 44.95 | | | | | | | | |
| NPV @ risk rate | | | | 0.00 | | | | | | | | |
| IRR | | | | 10.00% | | | | | | | | |
| **Projection** | | | | | | | | | | | | |
| Year | Age | lx | | qx | Premium | Initial Expenses | Maint Expenses | Death Claims | Maturity Claims | Interest | Reserves | Transfer |
| | | | | | BOY | BOY | BOY | EOY | EOY | EOY | EOY | EOY |
| 1 | 45 | 1.00000 | | 0.00264 | **844.39** | 80.00 | 10.00 | 26.37 | 0.00 | 30.18 | 828.17 | -69.98 |
| 2 | 46 | 0.99736 | | 0.00298 | 842.16 | 0.00 | 9.97 | 29.69 | 0.00 | 66.41 | 1716.71 | -19.62 |
| 3 | 47 | 0.99439 | | 0.00336 | 839.66 | 0.00 | 9.94 | 33.39 | 0.00 | 101.86 | 2625.54 | -10.66 |
| 4 | 48 | 0.99105 | | 0.00378 | 836.84 | 0.00 | 9.91 | 37.50 | 0.00 | 138.10 | 3554.54 | -1.47 |
| 5 | 49 | 0.98730 | | 0.00426 | 833.67 | 0.00 | 9.87 | 42.05 | 0.00 | 175.13 | 4503.50 | 7.92 |
| 6 | 50 | 0.98310 | | 0.00479 | 830.12 | 0.00 | 9.83 | 47.08 | 0.00 | 212.95 | 5472.13 | 17.53 |
| 7 | 51 | 0.97839 | | 0.00538 | 826.14 | 0.00 | 9.78 | 52.61 | 0.00 | 251.54 | 6460.08 | 27.35 |
| 8 | 52 | 0.97313 | | 0.00603 | 821.70 | 0.00 | 9.73 | 58.69 | 0.00 | 290.88 | 7466.87 | 37.37 |
| 9 | 53 | 0.96726 | | 0.00676 | 816.75 | 0.00 | 9.67 | 65.34 | 0.00 | 330.96 | 8491.95 | 47.61 |
| 10 | 54 | 0.96073 | | 0.00756 | 811.23 | 0.00 | 9.61 | 72.59 | 9534.69 | 371.74 | 0.00 | 58.04 |

- We need to increase the premium to ensure that we are earning the rate our shareholders require

  Premium 838.98 -> 844.39

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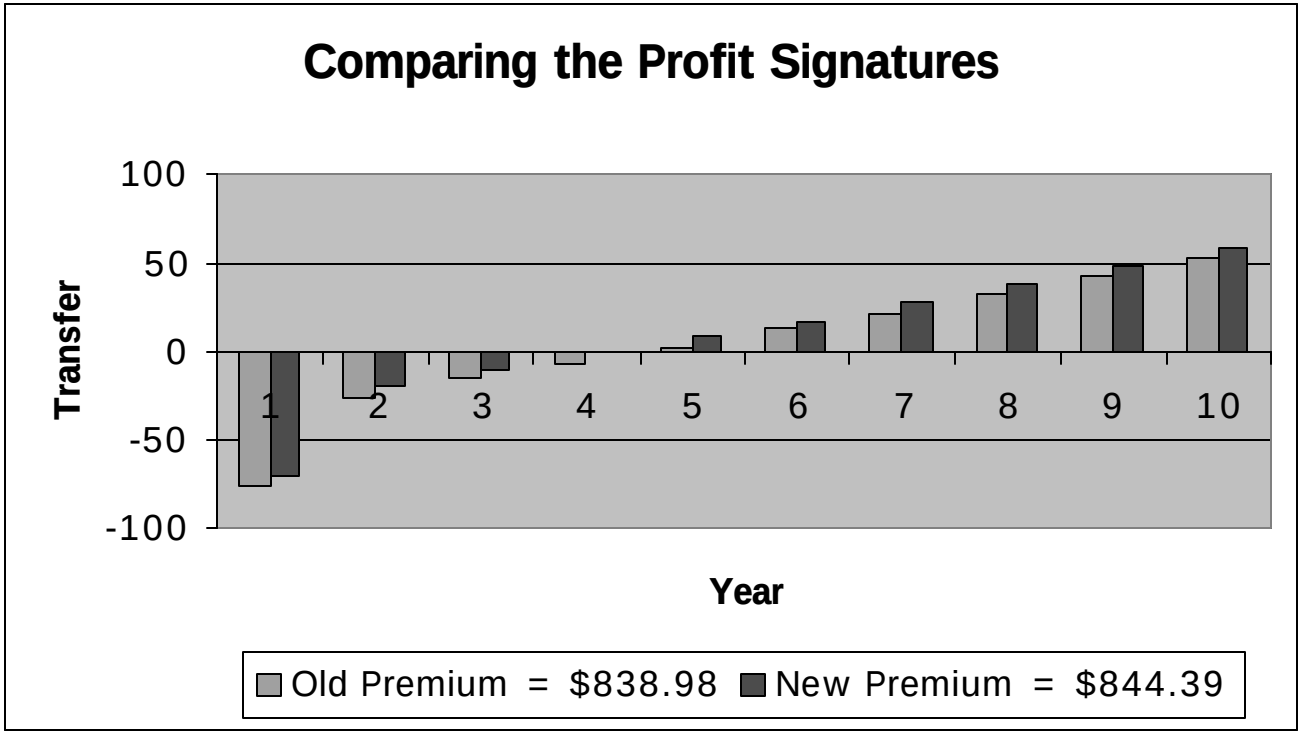
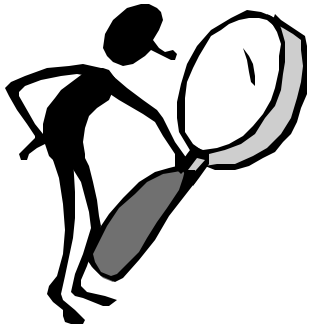
# Applying the Techniques (3)



# Sensitivity Testing



- Change one variable at a time to determine the effect on profitability
  - Determine the effect on profitability if assumptions do not turn out as expected
  - Determine which assumptions are the most crucial
  - Understand your product and the risks

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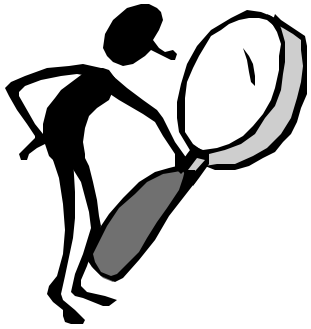
# Scenario Testing



- Similar to sensitivity analysis except that more than one variable is changed at a time
- Examine a set of reasonably realistic scenarios and determine the effect on profitability (eg a worse case scenario)
- In reality, some variables are linked (eg inflation and interest rates, interest rates and lapses, etc)

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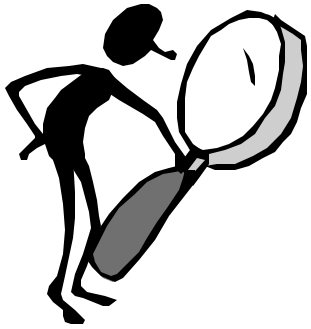
# Sensitivity Testing Example (1)



- Look at two different products
  - Endowment Insurance
  - Term Insurance (no maturity benefit)
- Same assumptions as previous example
- Premiums have been set to earn the shareholders required return on capital

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# Sensitivity Testing Example (2)

| Measures of Profitability | |
|---|---|
| NPV @ earned rate | 44.95 |
| NPV @ risk rate | 0.00 |
| IRR | 10.00% |

- Endowment Insurance

**Projection**

| Year | Age | lx | qx | Premium | Initial Expenses | Maint Expenses | Death Claims | Maturity Claims | Interest | Reserves | Transfer |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | BOY | BOY | BOY | EOY | EOY | EOY | EOY | EOY |
| 1 | 45 | 1.00000 | 0.00264 | **844.39** | 80.00 | 10.00 | 26.37 | 0.00 | 30.18 | 828.17 | -69.98 |
| 2 | 46 | 0.99736 | 0.00298 | 842.16 | 0.00 | 9.97 | 29.69 | 0.00 | 66.41 | 1716.71 | -19.62 |
| 3 | 47 | 0.99439 | 0.00336 | 839.66 | 0.00 | 9.94 | 33.39 | 0.00 | 101.86 | 2625.54 | -10.66 |
| 4 | 48 | 0.99105 | 0.00378 | 836.84 | 0.00 | 9.91 | 37.50 | 0.00 | 138.10 | 3554.54 | -1.47 |
| 5 | 49 | 0.98730 | 0.00426 | 833.67 | 0.00 | 9.87 | 42.05 | 0.00 | 175.13 | 4503.50 | 7.92 |
| 6 | 50 | 0.98310 | 0.00479 | 830.12 | 0.00 | 9.83 | 47.08 | 0.00 | 212.95 | 5472.13 | 17.53 |
| 7 | 51 | 0.97839 | 0.00538 | 826.14 | 0.00 | 9.78 | 52.61 | 0.00 | 251.54 | 6460.08 | 27.35 |
| 8 | 52 | 0.97313 | 0.00603 | 821.70 | 0.00 | 9.73 | 58.69 | 0.00 | 290.88 | 7466.87 | 37.37 |
| 9 | 53 | 0.96726 | 0.00676 | 816.75 | 0.00 | 9.67 | 65.34 | 0.00 | 330.96 | 8491.95 | 47.61 |
| 10 | 54 | 0.96073 | 0.00756 | 811.23 | 0.00 | 9.61 | 72.59 | 9534.69 | 371.74 | 0.00 | 58.04 |

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# Sensitivity Testing Example (3)

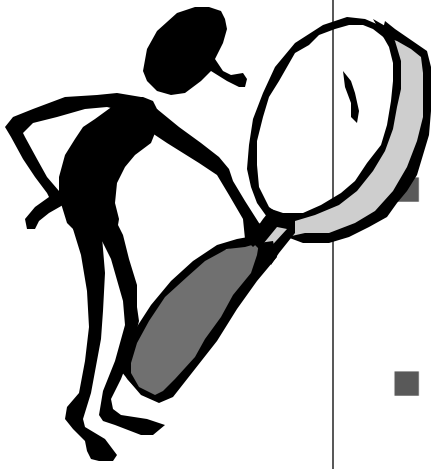
| Measures of Profitability | |
|---|---|
| NPV @ earned rate | 10.12 |
| NPV @ risk rate | -0.01 |
| IRR | 9.99% |

- Term Insurance

**Projection**

| Year | Age | lx | qx | Premium | Initial Expenses | Maint Expenses | Death Claims | Maturity Claims | Interest | Reserves | Transfer |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | BOY | BOY | BOY | EOY | EOY | EOY | EOY | EOY |
| 1 | 45 | 1.00000 | 0.00264 | **64.69** | 80.00 | 10.00 | 26.37 | 0.00 | -1.01 | -16.79 | -35.90 |
| 2 | 46 | 0.99736 | 0.00298 | 64.52 | 0.00 | 9.97 | 29.69 | 0.00 | 1.51 | 3.66 | 5.91 |
| 3 | 47 | 0.99439 | 0.00336 | 64.33 | 0.00 | 9.94 | 33.39 | 0.00 | 2.32 | 20.87 | 6.10 |
| 4 | 48 | 0.99105 | 0.00378 | 64.11 | 0.00 | 9.91 | 37.50 | 0.00 | 3.00 | 34.32 | 6.25 |
| 5 | 49 | 0.98730 | 0.00426 | 63.87 | 0.00 | 9.87 | 42.05 | 0.00 | 3.53 | 43.44 | 6.36 |
| 6 | 50 | 0.98310 | 0.00479 | 63.60 | 0.00 | 9.83 | 47.08 | 0.00 | 3.89 | 47.58 | 6.43 |
| 7 | 51 | 0.97839 | 0.00538 | 63.29 | 0.00 | 9.78 | 52.61 | 0.00 | 4.04 | 46.08 | 6.44 |
| 8 | 52 | 0.97313 | 0.00603 | 62.95 | 0.00 | 9.73 | 58.69 | 0.00 | 3.97 | 38.19 | 6.40 |
| 9 | 53 | 0.96726 | 0.00676 | 62.57 | 0.00 | 9.67 | 65.34 | 0.00 | 3.64 | 23.11 | 6.28 |
| 10 | 54 | 0.96073 | 0.00756 | 62.15 | 0.00 | 9.61 | 72.59 | 0.00 | 3.03 | 0.00 | 6.09 |

# Sensitivity Testing Example (4)



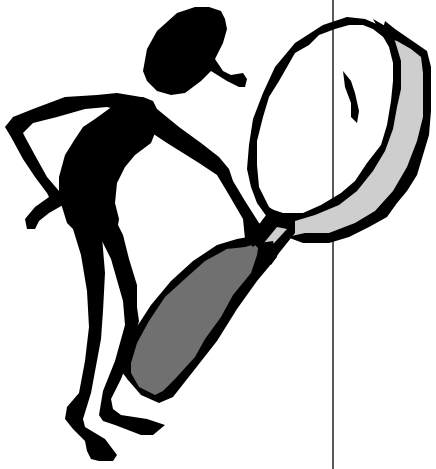
Endowment

- NPV @ earned rate
  = 44.95
- NPV @ risk rate
  = 0.00
- IRR
  = 10.00%

Term

- NPV @ earned rate
  = 10.12
- NPV @ risk rate
  = -0.01
- IRR
  = 9.99%

# Sensitivity Testing Example (5)



Sensitivity Tests

- Increase mortality by 5 %
- Decrease interest earning rate by 0.25 % (25 basis points)

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# Increase Mortality by 5%

- Endowment
- NPV @ earned rate
  = 37.15 (44.95)
- NPV @ risk rate
  = -6.20 (0.00)
- IRR
  = 8.89% (10.00%)

- Term
- NPV @ earned rate
  = -8.06 (10.12)
- NPV @ risk rate
  = -13.09 (-0.01)
- IRR
  = -1.71% (9.99%)

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# Decrease Interest Rate by 0.25%

- Endowment
- NPV @ earned rate
  = -47.75 (44.95)
- NPV @ risk rate
  = -64.41 (0.00)
- IRR
  = -3.68% (10.00%)

- Term
- NPV @ earned rate
  = 9.29 (10.12)
- NPV @ risk rate
  = -0.96 (-0.01)
- IRR
  = 9.26% (9.99%)

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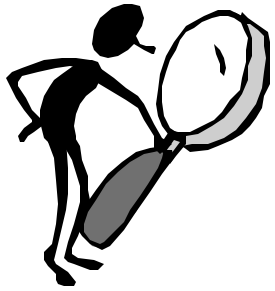
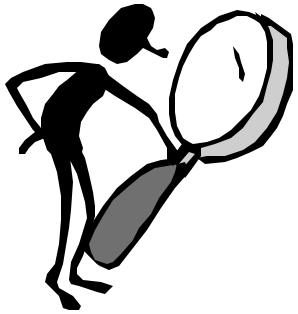
# Sensitivity Test Results (1)



- Term
  - Sensitive to changes in mortality, but not to changes in interest rates
  - It is important that you get your mortality assumptions right
  - Underwriting is very important
  - It is not so important where the money is invested

# Sensitivity Test Results (2)



- Endowment
  - Endowment policies are sensitive to changes in interest rates, but not changes in mortality
  - Mortality assumptions are not so important
  - It is very important where the money is invested

# Surrenders and Lapses (1)

- How were surrenders and lapses treated under traditional actuarial methods?
  - ANSWER: They were ignored.
    - The traditional actuarial approach was to have surrender values which were less than the policy value so that a profit was made on surrender
    - The effect was never quantified

# Surrenders and Lapses (2)

- What needs to be added
  - surrender value scale (possible cash flows)
  - surrender / lapse rates ($s_x$)
  - double decrement table (probabilities)
    $(aq)_x = q_x * (1 - \frac{1}{2} s_x)$
    $(as)_x = s_x * (1 - \frac{1}{2} q_x)$
    $(al)_{x+1} = (al)_x * (1 - s_x) * (1 - q_x)$
  - surrender claims become an extra cash flow item for calculating interest and transfers

# Surrenders and Lapses (3)

- Set the surrender basis equal (approximately) to the pricing basis
  - A67/70 mortality
  - 4 % interest
  - 80 Zillmer adjustment for initial expenses

# Surrenders and Lapses (4)

- Assumed surrender rate: 5% pa

| Measures of Profitability | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| NPV @ earned rate | | | | 40.06 | | | | | | | | | |
| NPV @ risk rate | | | | 3.74 | | | | | | | | | |
| IRR | | | | 10.86% | | | | | | | | | |
| **Projection** | | | | | | | | | | | | | |
| Year | Age | (al)x | (aq)x | (as)x | Premium | Initial Expenses | Maint Expenses | Death Claims | Maturity Claims | Surrender Claims | Interest | Reserves | Transfer |
| | | | | | BOY | BOY | BOY | EOY | EOY | EOY | EOY | EOY | EOY |
| 1 | 45 | 1.00000 | 0.00257 | 0.04993 | 844.39 | 80.00 | 10.00 | 25.71 | 0.00 | **37.68** | 30.18 | 786.77 | -65.59 |
| 2 | 46 | 0.94749 | 0.00290 | 0.04993 | 800.06 | 0.00 | 9.47 | 27.50 | 0.00 | **76.72** | 63.09 | 1549.33 | -13.12 |
| 3 | 47 | 0.89744 | 0.00327 | 0.04992 | 757.79 | 0.00 | 8.97 | 29.38 | 0.00 | **113.06** | 91.93 | 2251.08 | -3.45 |
| 4 | 48 | 0.84971 | 0.00369 | 0.04991 | 717.48 | 0.00 | 8.50 | 31.35 | 0.00 | **146.81** | 118.40 | 2895.20 | 5.11 |
| 5 | 49 | 0.80417 | 0.00415 | 0.04989 | 679.03 | 0.00 | 8.04 | 33.39 | 0.00 | **178.11** | 142.65 | 3484.72 | 12.60 |
| 6 | 50 | 0.76070 | 0.00467 | 0.04988 | 642.33 | 0.00 | 7.61 | 35.52 | 0.00 | **207.06** | 164.78 | 4022.52 | 19.12 |
| 7 | 51 | 0.71921 | 0.00524 | 0.04987 | 607.29 | 0.00 | 7.19 | 37.71 | 0.00 | **233.79** | 184.90 | 4511.31 | 24.72 |
| 8 | 52 | 0.67957 | 0.00588 | 0.04985 | 573.83 | 0.00 | 6.80 | 39.96 | 0.00 | **258.39** | 203.13 | 4953.67 | 29.46 |
| 9 | 53 | 0.64170 | 0.00659 | 0.04983 | 541.85 | 0.00 | 6.42 | 42.26 | 0.00 | **280.96** | 219.56 | 5352.05 | 33.39 |
| 10 | 54 | 0.60550 | 0.00737 | 0.04981 | 511.28 | 0.00 | 6.05 | 44.61 | 5708.77 | **301.61** | 234.29 | 0.00 | 36.58 |

# Surrenders and Lapses (5)

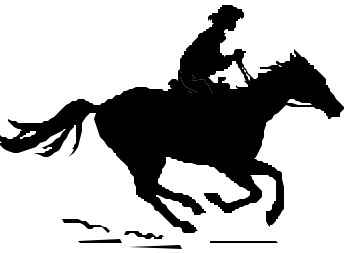
- Profitability increases slightly since surrenders release reserves earlier than otherwise expected
- Notice that if the surrender value basis is close to the pricing basis, then the addition of surrenders has little impact on profitability
- A more generous surrender value basis would lead to expected losses

# Surrenders and Lapses (6)

| Measures of Profitability | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| NPV @ earned rate | | | | 26.88 | | | | | | | |
| NPV @ risk rate | | | | -6.47 | | | | | | | |
| IRR | | | | 8.53% | | | | | | | |
| Projection | | | | | | | | | | | |
| Year | Age | (al)x | (aq)x | (as)x | Premium | Death Claims | Maturity Claims | Surrender Claims | Interest | Reserves | Transfer |
| | | | | | BOY | EOY | EOY | EOY | EOY | EOY | EOY |
| 1 | 45 | 1.00000 | 0.00257 | 0.04993 | 844.39 | 25.71 | 0.00 | **38.65** | 30.18 | 786.77 | -66.56 |
| 2 | 46 | 0.94749 | 0.00290 | 0.04993 | 800.06 | 27.50 | 0.00 | **78.40** | 63.09 | 1549.33 | -14.79 |
| 3 | 47 | 0.89744 | 0.00327 | 0.04992 | 757.79 | 29.38 | 0.00 | **115.20** | 91.93 | 2251.08 | -5.58 |
| 4 | 48 | 0.84971 | 0.00369 | 0.04991 | 717.48 | 31.35 | 0.00 | **149.19** | 118.40 | 2895.20 | 2.73 |
| 5 | 49 | 0.80417 | 0.00415 | 0.04989 | 679.03 | 33.39 | 0.00 | **180.51** | 142.65 | 3484.72 | 10.20 |
| 6 | 50 | 0.76070 | 0.00467 | 0.04988 | 642.33 | 35.52 | 0.00 | **209.31** | 164.78 | 4022.52 | 16.88 |
| 7 | 51 | 0.71921 | 0.00524 | 0.04987 | 607.29 | 37.71 | 0.00 | **235.69** | 184.90 | 4511.31 | 22.81 |
| 8 | 52 | 0.67957 | 0.00588 | 0.04985 | 573.83 | 39.96 | 0.00 | **259.80** | 203.13 | 4953.67 | 28.05 |
| 9 | 53 | 0.64170 | 0.00659 | 0.04983 | 541.85 | 42.26 | 0.00 | **281.73** | 219.56 | 5352.05 | 32.62 |
| 10 | 54 | 0.60550 | 0.00737 | 0.04981 | 511.28 | 44.61 | 5708.77 | **301.61** | 234.29 | 0.00 | 36.58 |

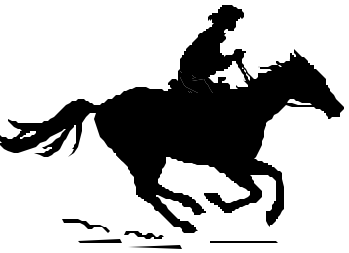
- A more generous surrender basis
  - 3.5 % interest (instead of 4.0 %)

# Adding a Rider (1)



- Example
  - Total and Permanent Disablement (TPD) benefit - level premiums
  - The TPD benefit is an acceleration of 50 % of the death benefit
  - Incidence Rate ($i_x$)
    1 per 1000 at age 45
    increasing by 0.5 each year to age 54

# Adding a Rider (2)



- What needs to be added
  - TPD Benefit (50% of Sum Insured) (possible cash flow)
  - TPD incidence rates ($i_x$) (probabilities) double decrement table (ignoring surrenders)
    $(aq)_x = q_x * (1 - ½ * 50\% \, i_x)$
    $(ai)_x = 50\% \, i_x * (1 - ½ \, q_x)$
    $(al)_{x+1} = (al)_x * (1 - 50\% \, i_x) * (1 - q_x)$
  - TPD claims become an extra cash flow item for calculating interest and transfers
  - Reserves should allow for TPD claims

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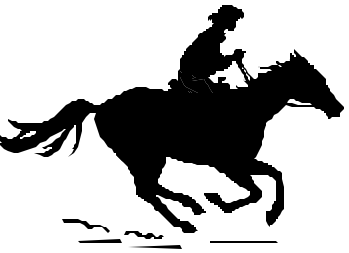
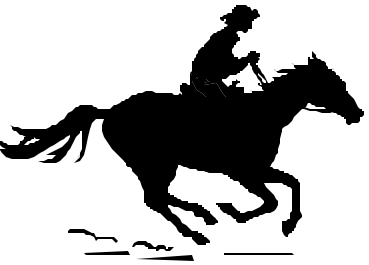
# Adding a Rider (3)



- In practice
  - because $(1 - \frac{1}{2} q_x)$ and $(1 - \frac{1}{2} * 50\% i_x)$ are so close to 1, this adjustment is often ignored
  - $(q_x + 50\% i_x)$ is often treated as a *single decrement*
  - this is conservative as it *overestimates* mortality and TPD incidence

# Adding a Rider (4)



- For our example
  - Including the double decrement adjustment, we get a premium of 850.07
  - Ignoring the double decrement adjustment, we get a premium of 850.13
  - Not much difference

# Adding a Rider (5)

- Ignoring the double decrement adjustment

| Measures of Profitability | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| NPV @ earned rate | | | | 44.64 | | | | | | | | | | |
| NPV @ risk rate | | | | 0.00 | | | | | | | | | | |
| IRR | | | | 10.00% | | | | | | | | | | |
| **Projection** | | | | | | | | | | | | | | |
| Year | Age | lx | qx | ix | Premium | Initial Expenses | Maint Expenses | Death Claims | Maturity Claims | TPD Claims | Interest | Reserves | Transfer | |
| | | | | | BOY | BOY | BOY | EOY | EOY | EOY | EOY | EOY | EOY | |
| 1 | 45 | 1.00000 | 0.00264 | 0.00050 | **850.13** | 80.00 | 10.00 | 26.37 | 0.00 | **5.00** | 30.41 | 828.99 | -69.82 | |
| 2 | 46 | 0.99686 | 0.00298 | 0.00075 | 847.47 | 0.00 | 9.97 | 29.68 | 0.00 | **7.48** | 66.66 | 1715.43 | -19.44 | |
| 3 | 47 | 0.99315 | 0.00336 | 0.00100 | 844.31 | 0.00 | 9.93 | 33.35 | 0.00 | **9.93** | 101.99 | 2618.99 | -10.47 | |
| 4 | 48 | 0.98882 | 0.00378 | 0.00125 | 840.63 | 0.00 | 9.89 | 37.42 | 0.00 | **12.36** | 137.99 | 3539.26 | -1.32 | |
| 5 | 49 | 0.98384 | 0.00426 | 0.00150 | 836.40 | 0.00 | 9.84 | 41.90 | 0.00 | **14.76** | 174.63 | 4475.76 | 8.03 | |
| 6 | 50 | 0.97818 | 0.00479 | 0.00175 | 831.58 | 0.00 | 9.78 | 46.84 | 0.00 | **17.12** | 211.90 | 5427.95 | 17.55 | |
| 7 | 51 | 0.97178 | 0.00538 | 0.00200 | 826.14 | 0.00 | 9.72 | 52.25 | 0.00 | **19.44** | 249.78 | 6395.22 | 27.25 | |
| 8 | 52 | 0.96461 | 0.00603 | 0.00225 | 820.05 | 0.00 | 9.65 | 58.18 | 0.00 | **21.70** | 288.22 | 7376.85 | 37.12 | |
| 9 | 53 | 0.95662 | 0.00676 | 0.00250 | 813.26 | 0.00 | 9.57 | 64.62 | 0.00 | **23.92** | 327.22 | 8372.06 | 47.16 | |
| 10 | 54 | 0.94777 | 0.00756 | 0.00275 | 805.73 | 0.00 | 9.48 | 71.61 | 9380.02 | **26.06** | 366.73 | 0.00 | 57.36 | |

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# Model Office (1)

- In all of our examples
  - we considered only one particular policy (male, age 45, 10 year endowment)
  - in reality, there are many different policies with many different parameters (age, sex, smoker, different policy terms, different policy types, etc)

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# Model Office (2)

- In practice
  - we would develop assumptions about the volume and mix of business (by sex, age, smoker, policy type, policy term, etc)
  - we would use cash flow pricing techniques to examine a range of "model points"
  - we would multiply the per policy cash flows by the expected volume for each model point
  - aggregate all the cash flows into a model office

# Model Office (3)

- This will allow us to
  - project expected capital injections and releases, which will allow us to plan for future capital needs
  - cross-subsidise between different market segments (age, sex, policy-type) and ensure that we are still profitable overall
  - understand the sensitivity of our whole portfolio to changes in experience assumptions or changes in the volume or mix of business
  - monitor emerging experience against our assumptions

# New Business Projections (1)

- Example
  - one policy type only
    (male, age 45, 10 year endowment,
    no surrenders or rider benefits)
  - new business projections

| | |
|---|---|
| 1998 | 100 policies |
| 1999 | 200 policies |
| 2000 | 300 policies |
| thereafter | 200 policies each year |

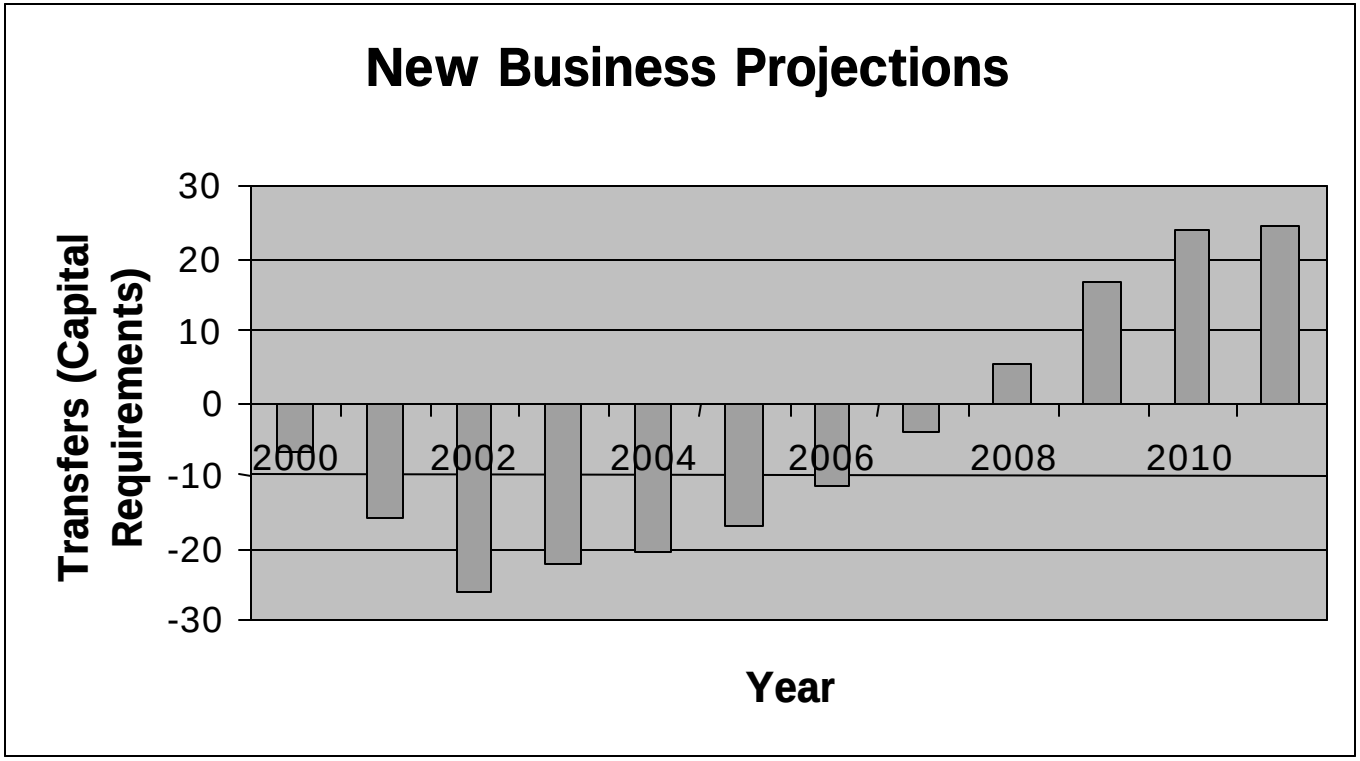
# New Business Projections (2)

- If initial capital strains are high and new business levels are high, negative transfers will continue for some time

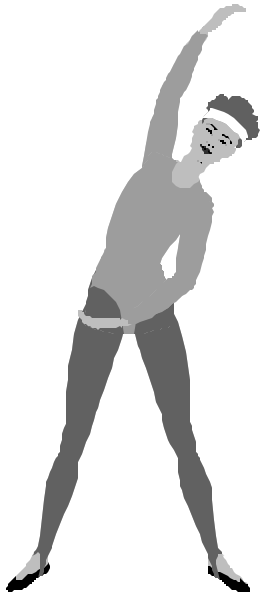
**New Business Projections**
**Capital Transfers**

| | Year of Issue | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 2000 | 2001 | 2002 | 2003 | 2004 | 2005 | 2006 | 2007 | 2008 | 2009 | 2010 | 2011 | Total |
| | 100 | 200 | 300 | 200 | 200 | 200 | 200 | 200 | 200 | 200 | 200 | 200 | |
| 2000 | -7.0 | | | | | | | | | | | | **-7.0** |
| 2001 | -2.0 | -14.0 | | | | | | | | | | | **-16.0** |
| 2002 | -1.1 | -3.9 | -21.0 | | | | | | | | | | **-26.0** |
| 2003 | -0.1 | -2.1 | -5.9 | -14.0 | | | | | | | | | **-22.2** |
| 2004 | 0.8 | -0.3 | -3.2 | -3.9 | -14.0 | | | | | | | | **-20.6** |
| 2005 | 1.8 | 1.6 | -0.4 | -2.1 | -3.9 | -14.0 | | | | | | | **-17.2** |
| 2006 | 2.7 | 3.5 | 2.4 | -0.3 | -2.1 | -3.9 | -14.0 | | | | | | **-11.7** |
| 2007 | 3.7 | 5.5 | 5.3 | 1.6 | -0.3 | -2.1 | -3.9 | -14.0 | | | | | **-4.3** |
| 2008 | 4.8 | 7.5 | 8.2 | 3.5 | 1.6 | -0.3 | -2.1 | -3.9 | -14.0 | | | | **5.2** |
| 2009 | 5.8 | 9.5 | 11.2 | 5.5 | 3.5 | 1.6 | -0.3 | -2.1 | -3.9 | -14.0 | | | **16.7** |
| 2010 | | 11.6 | 14.3 | 7.5 | 5.5 | 3.5 | 1.6 | -0.3 | -2.1 | -3.9 | -14.0 | | **23.6** |
| 2011 | | | 17.4 | 9.5 | 7.5 | 5.5 | 3.5 | 1.6 | -0.3 | -2.1 | -3.9 | -14.0 | **24.6** |

# New Business Projections (3)



# Extensions (1)



- The basic cash flow model
  - columns of expected cash flows and intermediate calculations
  - expected cash flows are derived by multiplying a possible cash flow by the probability that the cash flow occurs
  - transfers - expected capital requirements and releases
  - measures of profitability

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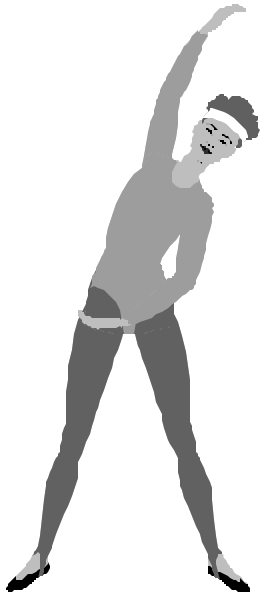
# Extensions (2)



- Extra cash flow columns can be added in the same way
  - additional benefits and riders
  - bonuses and distributions of surplus
  - reinsurance cash flows
  - taxation
  - etc

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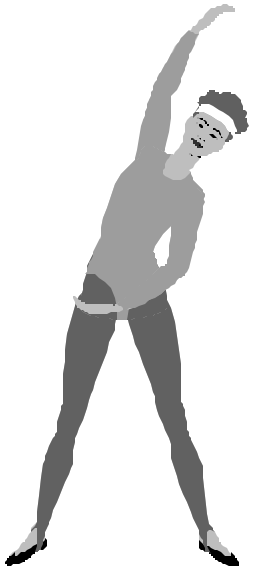
# Extensions (3)



- Further enhancements
  - multiple state models
    (eg for disability income products)
  - stochastic models
    (eg to calculate probabilities and distributions)
  - asset/liability models
    (considering the interaction between assets and liabilities)

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# Summary - Cash Flow Methods

- Easy
  - cash flows shown explicitly, easy to construct and understand, etc.
- Versatile
  - complex benefits, surrenders, etc
- Effective
  - profitability, sensitivity testing, capital requirements, etc.

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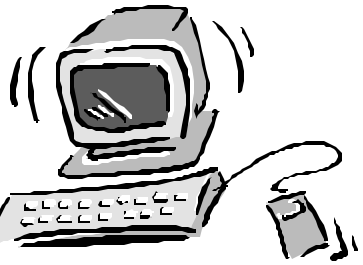
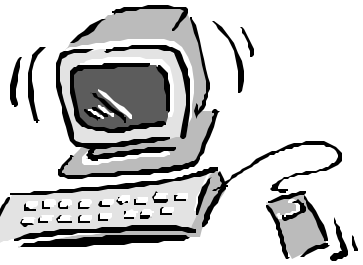
# Questions?

# Some Useful Spreadsheet Functions (1)



- Microsoft Excel
  - VLOOKUP(value; table; column; [range])
  - IRR(values; guess)
  - NPV(rate; values)
  - Tools/Goal Seek or Tools/Solver

# Some Useful Spreadsheet Functions (2)



- Lotus 1-2-3
  - @VLOOKUP(x; range; column-offset)
  - @IRR(guess; range)
  - @NPV(interest; range; [type])
  - Range/Analyse/Backsolver or Range/Analyse/Solver

# A few more hints

- In a spreadsheet model, try to use named variables and ranges so that variables can be easily changed
- In practice, claims would usually be assumed to occur in the middle of the year rather than the end of the year - this affects the interest column
- Greater accuracy can be achieved by performing monthly rather than yearly projections
- Spreadsheets are useful for simple models or pilot models, but any significant work should make use of a computer program